1. Since we form an open top box, this means we have:

The surface area $A$ is given by

$$A = xy + 2xz + 2yz \quad (*)$$

We are given that the volume of the box is $3 \text{m}^3$ therefore $xyz = 3$. Making $z$ the subject gives

$$z = \frac{3}{xy} \quad (†)$$

Substituting this into (*) yields

$$A = xy + 2x \frac{3}{xy} + 2y \frac{3}{xy}$$

$$= xy + \frac{6}{y} + \frac{6}{x}$$

Thus we have our required result.

To find which dimensions give minimum surface area we first find the stationary points of $A$. These are determined by partially differentiating the derived result and equating them to zero. We have

$$A = xy + \frac{6}{y} + \frac{6}{x} = xy + 6y^{-1} + 6x^{-1}$$

$$\frac{\partial A}{\partial x} = y + 0 - 6x^{-2} = 0 \quad \text{implies} \quad y = \frac{6}{x^2}$$

$$\frac{\partial A}{\partial y} = x - 6y^{-2} = 0 \quad \text{implies} \quad x - \frac{6}{y^2} = 0$$

Substituting $y = \frac{6}{x^2}$ into the bottom equation $x - \frac{6}{y^2} = 0$ gives

$$x - \frac{6}{\left(\frac{6}{x^2}\right)^2} = x - \frac{6}{36/x^4}$$

$$= x - \frac{x^4}{6} = x \left(6 - x^3\right) = 0$$

Solving the last equation $x \left(6 - x^3\right) = 0$ we have
\[ x = 0, \quad x = 6^{1/3} \]

We cannot have \( x = 0 \) because this means we will not have a box. Hence \( x = 6^{1/3} = 1.8171 \).

Substituting this \( x = 6^{1/3} \) into the above equation \( y = \frac{6}{x^2} \) yields that

\[ y = \frac{6}{(6^{1/3})^2} = \frac{6^1}{6^{2/3}} = 6^{1/3} \]

Hence \( y = x = 6^{1/3} = 1.8171 \). We need to check that these \( x \) and \( y \) values do indeed give us minimum surface area. How?

By using the second derivative test. We have

\[
\frac{\partial^2 A}{\partial x^2} = y - 6x^{-2}
\]

\[
\frac{\partial^2 A}{\partial x^2} = -6x^{-3} = \frac{12}{x^3}
\]

Similarly (or by examining the symmetry of the expression) from \( \frac{\partial A}{\partial y} = x - 6y^{-2} \) we obtain

\[
\frac{\partial^2 A}{\partial y^2} = \frac{12}{y^3}. \quad \text{Which other second derivative do we need to find?}
\]

The mixed partial derivative \( \frac{\partial^2 A}{\partial x \partial y} \). This partial derivative is given by

\[
\frac{\partial^2 A}{\partial x \partial y} = \frac{\partial}{\partial x}\left( \frac{\partial A}{\partial y} \right) = \frac{\partial}{\partial x}\left( x - 6y^{-2} \right) = 1 - 0 = 1
\]

Putting all these \( \frac{\partial^2 A}{\partial x^2} = \frac{12}{x^3}, \quad \frac{\partial^2 A}{\partial y^2} = \frac{12}{y^3} \) and \( \frac{\partial^2 A}{\partial x \partial y} = 1 \) into formula (15.11)

\[
\left( \frac{\partial^2 A}{\partial x^2} \right) \left( \frac{\partial^2 A}{\partial y^2} \right) - \left( \frac{\partial^2 A}{\partial x \partial y} \right)^2 = \left( \frac{12}{x^3} \right) \left( \frac{12}{y^3} \right) - 1^2 = \frac{144}{x^3 y^3} - 1 \quad (**)
\]

Substituting \( y = x = 6^{1/3} \) into this (**) gives

\[
\frac{144}{(6^{1/3})^3 (6^{1/3})^3} - 1 = \frac{144}{36} - 1 > 0 \quad \text{[Positive]}
\]

We know from formula (15.11) that we have a minimum or maximum. Putting \( x = 6^{1/3} \) into

\[
\frac{\partial^2 A}{\partial x^2} = \frac{12}{x^3} \quad \text{gives} \quad \frac{\partial^2 A}{\partial x^2} = \frac{12}{6^3/2} = 2 > 0. \quad \text{Hence we have a minimum when} \quad y = x = 6^{1/3}.
\]

What is the value of \( z \)?

Substituting \( y = x = 6^{1/3} \) into above (†) equation which is \( z = \frac{3}{xy} \) gives

\[
z = \frac{3}{xy} = \frac{3}{6^{1/3}6^{1/3}} = \frac{3}{6^{2/3}} = 0.9086
\]

Our dimensions for minimum surface area are \( y = x = 1.817 \) and \( z = 0.909 \) (correct to 3dp).
2. We need to find the critical points of the given function \( f(x, y) = y^3 - x^3 + 3xy + 1 \).

To determine the stationary points we need to partially differentiate \( f \):

\[
\frac{\partial f}{\partial x} = 0 - 3x^2 + 3y = 0 \quad \text{implying} \quad y = x^2
\]

\[
\frac{\partial f}{\partial y} = 3y^2 - 0 + 3x = 0 \quad \text{implying} \quad 3y^2 + 3x = 0
\]

Substituting \( y = x^2 \) into the bottom equation \( 3y^2 + 3x = 0 \) gives

\[
3(x^4) + 3x = 3x + x(x^3 + 1) = 0
\]

From this we have \( x = 0, \ x = -1 \). Substituting these into \( y = x^2 \) yields that \( y = 0, \ y = 1 \) respectively.

Our stationary points are \((0, 0)\) and \((-1, 1)\). We need to decide whether these points are maximum, minimum or saddle points. This means we need to differentiate again:

\[
\frac{\partial^2 f}{\partial x^2} = -3x^2 + 3y
\]

\[
\frac{\partial^2 f}{\partial x \partial y} = -6x
\]

\[
\frac{\partial^2 f}{\partial y^2} = 3y^2 + 3x
\]

The mixed partial derivative is given by

\[
\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial x} \left( 3y^2 + 3x \right) = 3
\]

Substituting these \( \frac{\partial^2 f}{\partial x^2} = -6x \), \( \frac{\partial^2 f}{\partial y^2} = 3y^2 + 3x \) and \( \frac{\partial^2 f}{\partial x \partial y} = 3 \) into formula (15.11):

\[
\left( \frac{\partial^2 A}{\partial x^2} \right)^2 - \left( \frac{\partial^2 A}{\partial y^2} \right)^2 = (-6x)(6y) - 3^2 = -36xy - 9 \quad (*)
\]

Substituting the stationary point \((0, 0)\) which means that \( x = 0, \ y = 0 \) into (*):

\[
\left( \frac{\partial^2 A}{\partial x^2} \right)^2 - \left( \frac{\partial^2 A}{\partial y^2} \right)^2 = -36(0)(0) - 9 = -9 < 0
\]

The stationary point \((0, 0)\) is a saddle point.

Next we test the other stationary point \((-1, 1)\) which means that \( x = -1, \ y = 1 \). Substituting this into (*) gives

\[
\left( \frac{\partial^2 A}{\partial x^2} \right)^2 - \left( \frac{\partial^2 A}{\partial y^2} \right)^2 = -36(-1)(1) - 9 = 36 - 9 > 0
\]

Thus \((-1, 1)\) is a maximum or minimum. Since
\[ \frac{\partial^2 f}{\partial x^2} = -6x = -6(-1) = 6 > 0 \]

therefore \((-1, 1)\) is a minimum.

We have \((0, 0)\) is a saddle point and \((-1, 1)\) is a minimum.

3. We need to find the values of \(x\) and \(y\) such that \(C = 4xy + \frac{72}{y} + \frac{48}{x}\) is a minimum. \textit{How?}

Apply partial differentiation and equate to zero to get the stationary points first:

\[ C = 4xy + \frac{72}{y} + \frac{48}{x} = 4xy + 72y^{-1} + 48x^{-1} \]

\[ \frac{\partial C}{\partial x} = 4y + 0 - 48x^{-2} = 0 \quad \text{implies} \quad 4y = 48x^{-2} = \frac{48}{x^2} \]

\[ \frac{\partial C}{\partial y} = 4x - 72y^{-2} + 0 = 0 \quad \text{implies} \quad 4x = \frac{72}{y^2} \]

Dividing the last two equations by 4 gives

\[ y = \frac{12}{x^2}, \quad x = \frac{18}{y^2} \]

Substituting \(y = \frac{12}{x^2}\) into \(x = \frac{18}{y^2}\) gives

\[ x = \frac{18}{(12/x^2)^2} = \frac{18x^4}{144} \]

\[ 144x - 18x^4 = 0 \]

\[ 18x(8 - x^3) = 0 \]

Solving the last equation gives \(x = 0, x = 2\). We are given that \(x \neq 0\) therefore \(x = 2\).

Substituting this \(x = 2\) into \(y = \frac{12}{x^2}\) gives \(y = \frac{12}{2^2} = 3\). Thus \((2, 3)\) is a stationary point.

We need to show that this stationary point is indeed a minimum. Determining the second partial derivatives

\[ \frac{\partial C}{\partial x} = 4y - 48x^{-2} \]

\[ \frac{\partial^2 C}{\partial x^2} = 98x^{-3} \]

\[ \frac{\partial C}{\partial y} = 4x - 72y^{-2} \]

\[ \frac{\partial^2 C}{\partial y^2} = 144y^{-3} \]

The mixed partial derivative is given by

\[ \frac{\partial^2 C}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial C}{\partial y} \right) = \frac{\partial}{\partial x} \left( 4x - 72y^{-2} \right) = 4 \]

Putting all these \(\frac{\partial^2 C}{\partial x^2} = 98x^{-3}, \frac{\partial^2 C}{\partial y^2} = 144y^{-3}\) and \(\frac{\partial^2 C}{\partial x \partial y} = 4\) into formula (15.11) yields
\[
\left( \frac{\partial^2 C}{\partial x^2} \right) \left( \frac{\partial^2 C}{\partial y^2} \right) - \left( \frac{\partial^2 C}{\partial x \partial y} \right)^2 = \left( 98x^{-3} \right) \left( 144y^{-3} \right) - 4^2 = \frac{98 \times 144}{x^3 y^3} - 16
\]

Substituting \( x = 2 \) and \( y = 3 \) into the above gives
\[
\left( \frac{\partial^2 C}{\partial x^2} \right) \left( \frac{\partial^2 C}{\partial y^2} \right) - \left( \frac{\partial^2 C}{\partial x \partial y} \right)^2 = \frac{98 \times 144}{2^3 3^3} - 16 > 0
\]

We have a maximum or minimum at \( x = 2 \) and \( y = 3 \). Applying
\[
\frac{\partial^2 C}{\partial x^2} = 98x^{-3} = \frac{98}{x^3}
\]

Putting \( x = 2 \) into \( \frac{\partial^2 C}{\partial x^2} = \frac{98}{x^3} \) gives a positive answer therefore we have a minimum at \( x = 2 \) and \( y = 3 \).

4. We are given that \( k = 4\pi^2 \frac{m}{T^2} \). Note that \( k \) is function of \( m \) and \( T \) therefore
\[
\Delta k \cong \frac{\partial k}{\partial m} \Delta m + \frac{\partial k}{\partial T} \Delta T \quad (*)
\]

Next we determine each component of (*) . From \( k = 4\pi^2 \frac{m}{T^2} \) we have
\[
\frac{\partial k}{\partial m} = \frac{4\pi^2}{T^2}, \quad \frac{\partial k}{\partial T} = (-2)4\pi^2 mT^{-3} = (-2)4\pi^2 \frac{m}{T^3}
\]

What is \( \Delta m \) equal to?
\[
\pm 1.5\% \text{ of } m = \pm 1.5 \times 100 m = \pm 0.015m. \text{ This means that } \Delta m = \pm 0.015m.
\]

What is \( \Delta T \) equal to?
\[
\pm 2\% \text{ of } T = \pm 2 \times 100 - T = \pm 0.02T. \text{ We have } \Delta T = \pm 0.02T.
\]

Substituting \( \frac{\partial k}{\partial m} = \frac{4\pi^2}{T^2}, \frac{\partial k}{\partial T} = (-2)4\pi^2 \frac{m}{T^3}, \Delta m = \pm 0.015m \) and \( \Delta T = \pm 0.02T \) into (*) gives
\[
\Delta k \cong \frac{\partial k}{\partial m} \Delta m + \frac{\partial k}{\partial T} \Delta T
\]
\[
= \frac{4\pi^2}{T^2} (-0.015m) + (-2)4\pi^2 \frac{m}{T^3} (0.02T)
\]
\[
= \frac{4\pi^2 m}{T^2} (0.015) - \frac{4\pi^2 m}{T^3} (0.02)
\]
\[
= \frac{4\pi^2 m}{T^2} [(0.015) - (0.02)]
\]
\[
= k [(0.015) - (0.02)] \quad \text{[Because } k = \frac{4\pi^2 m}{T^2} \text{]}.
\]
What is the maximum error in measuring $k$?
The maximum error occurs when the signs $\pm$ in the above are different:

$$\Delta k \approx k\left[ (+0.015) - (-0.02) \right]$$
$$= k \left[ 0.035 \right]$$

The largest percentage error in the measurement of $k$ is 3.5%.

5. (a) We need to partial differentiate the given function $u = 3x^2 + 6x^2y - 4xy^2 - y^3$:

$$u = 3x^2 + 6x^2y - 4xy^2 - y^3$$
$$\frac{\partial u}{\partial x} = 6x + 12xy - 4y^2$$
$$\frac{\partial^2 u}{\partial x^2} = 6 + 12y$$

Similarly we have

$$u = 3x^2 + 6x^2y - 4xy^2 - y^3$$
$$\frac{\partial u}{\partial y} = 6x^2 - 8xy - 3y^2$$
$$\frac{\partial^2 u}{\partial y^2} = -8x - 6y$$

The mixed partial derivative is given by

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial y} \right) = \frac{\partial}{\partial x} \left[ 6x^2 - 8xy - 3y^2 \right]$$
$$= 12x - 8y$$

(b) We need to show that for the given $u$ we have $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = c^2 \frac{\partial u}{\partial t}$. We have

$$u = \sin(2x)\sin(4y)e^{-st}$$
$$\frac{\partial u}{\partial x} = 2\cos(2x)\sin(4y)e^{-st}$$
$$\frac{\partial^2 u}{\partial x^2} = -4\sin(2x)\sin(4y)e^{-st}$$

Similarly we have

$$u = \sin(2x)\sin(4y)e^{-st}$$
$$\frac{\partial u}{\partial y} = 4\sin(2x)\cos(4y)e^{-st}$$
$$\frac{\partial^2 u}{\partial y^2} = -16\sin(2x)\sin(4y)e^{-st}$$

Additionally we have

$$u = \sin(2x)\sin(4y)e^{-st}$$
$$\frac{\partial u}{\partial t} = -5\sin(2x)\sin(4y)e^{-st}$$
Substituting the above $\frac{\partial^2 u}{\partial x^2} = -4\sin(2x)\sin(4y)e^{-5t}$ and $\frac{\partial^2 u}{\partial y^2} = -16\sin(2x)\sin(4y)e^{-5t}$ into the left hand side of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = c^2 \frac{\partial u}{\partial t}$ gives

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -4\sin(2x)\sin(4y)e^{-5t} - 16\sin(2x)\sin(4y)e^{-5t}$$

$$= -20\sin(2x)\sin(4y)e^{-5t}$$

$$= 4\left[-5\sin(2x)\sin(4y)e^{-5t}\right]$$

$$= 4\frac{\partial u}{\partial t} \quad \text{[Because } \frac{\partial u}{\partial t} = -5\sin(2x)\sin(4y)e^{-5t}]$$

Hence $c^2 = 4$ which gives $c = 2$.

6. (a) (i) We are given $z = x^3 + 5x^2y + 2y^3$. We need to find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y\partial x}$:

$$z = x^3 + 5x^2y + 2y^3$$

$$\frac{\partial z}{\partial x} = 3x^2 + 10xy$$

$$\frac{\partial^2 z}{\partial x^2} = 6x + 10y$$

$$\frac{\partial z}{\partial y} = 5x^2 + 6y^2$$

The mixed partial derivative can be determined as follows:

$$\frac{\partial^2 z}{\partial y\partial x} = \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial x} \right) = \frac{\partial}{\partial y} \left( 3x^2 + 10xy \right) = 10x$$

(ii) Similarly for $z = e^x \cos(y)$ we need to find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y\partial x}$:

$$z = e^x \cos(y)$$

$$\frac{\partial z}{\partial x} = e^x \cos(y)$$

$$\frac{\partial^2 z}{\partial x^2} = e^x \cos(y)$$

$$\frac{\partial z}{\partial y} = -e^x \sin(y)$$

$$\frac{\partial^2 z}{\partial y\partial x} = \frac{\partial}{\partial y} \left( \frac{\partial z}{\partial x} \right) = \frac{\partial}{\partial y} \left[ e^x \cos(y) \right] = -e^x \sin(y)$$

(b) We are given $z = e^{px} (x \cos(y) - y \sin(y))$. We need to find a value of $p$ which satisfies the following:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$
\[ z = e^{px} \left( x \cos(y) - y \sin(y) \right) \]

\[ \frac{\partial z}{\partial x} = pe^{px} \left( x \cos(y) - y \sin(y) \right) + e^{px} \cos(y) \]

\[ = e^{px} \left[ px \cos(y) - py \sin(y) + \cos(y) \right] \]

\[ \frac{\partial^2 z}{\partial x^2} = pe^{px} \left[ px \cos(y) - py \sin(y) + \cos(y) \right] + e^{px} p \cos(y) \]

Similarly we have

\[ z = e^{px} \left( x \cos(y) - y \sin(y) \right) \]

\[ \frac{\partial z}{\partial y} = e^{px} \left( -x \sin(y) - \left[ \left(1 - y^2\right) + y \cos(y) \right] \right) \]

\[ = e^{px} \left( -x \sin(y) - \sin(y) - y \cos(y) \right) \]

\[ \frac{\partial^2 z}{\partial y^2} = e^{px} \left( -x \cos(y) - \cos(y) - \left[ \left(1 - y^2\right) \right] \right) \]

\[ = e^{px} \left( -x \cos(y) - \cos(y) - \cos(y) + y \sin(y) \right) \]

\[ = e^{px} \left( -x \cos(y) - 2 \cos(y) + y \sin(y) \right) \]

Adding the two second derivatives

\[ \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = pe^{px} \left[ px \cos(y) - py \sin(y) + 2 \cos(y) \right] \]

\[ + e^{px} \left[ -x \cos(y) - 2 \cos(y) + y \sin(y) \right] \]

\[ = e^{px} \left[ p^2 x \cos(y) - x \cos(y) - p^2 y \sin(y) + y \sin(y) \right] \]

\[ + 2 p \cos(y) - 2 \cos(y) \]

\[ = e^{px} \left[ \left( p^2 - 1 \right) x \cos(y) + \left(1 - p^2\right) y \sin(y) + 2 \left(p - 1\right) \cos(y) \right] \]

Clearly this is zero when \( p = 1 \) because all the terms in the square bracket are zero at \( p = 1 \).

(c) We need to find the stationary points of \( f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x \):

\[ f = 2x^3 + 6xy^2 - 3y^3 - 150x \]

\[ \frac{\partial f}{\partial x} = 6x^2 + 6y^2 - 150 = 0 \quad \Rightarrow \quad x^2 + y^2 = \frac{150}{6} = 25 \]

\[ \frac{\partial f}{\partial y} = 12xy - 9y^2 = 0 \quad \Rightarrow \quad 3y(4x - 3y) = 0 \]

From the bottom equation we have \( y = 0 \) or \( 4x - 3y = 0 \) \( \Rightarrow \quad x = \frac{3y}{4} \).

Substituting \( y = 0 \) into the first equation \( x^2 + y^2 = 25 \) gives \( x^2 = 25 \) \( \Rightarrow \quad x = \pm 5 \).

Substituting \( x = \frac{3y}{4} \) into the first equation \( x^2 + y^2 = 25 \) gives...
\[ x^2 + y^2 = \left( \frac{3y}{4} \right)^2 + y^2 \]
\[ = \frac{9y^2}{16} + y^2 = \frac{9y^2 + 16y^2}{16} = \frac{25y^2}{16} = 25 \]
\[ y^2 = 16 \text{ gives } y = \pm 4 \]

If \( y = 4 \) then \( x = \frac{3y}{4} = \frac{3 \times 4}{4} = 3 \). Similarly if \( y = -4 \) then \( x = -3 \).

We have four stationary points \((5, 0), (-5, 0), (3, 4)\) and \((-3, -4)\). We need to check the nature of each of these points. This means we need to find the second partial derivatives:
\[ f = 2x^3 + 6xy^2 - 3y^3 - 150x \]
\[ \frac{\partial f}{\partial x} = 6x^2 + 6y^2 - 150 \]
\[ \frac{\partial^2 f}{\partial x^2} = 12x \]
\[ \frac{\partial f}{\partial y} = 12xy - 9y^2 \]
\[ \frac{\partial^2 f}{\partial y^2} = 12x - 18y \]

We also need to determine the mixed partial derivative
\[ \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) \]
\[ = \frac{\partial}{\partial x} (12xy - 9y^2) = 12y \]

Substituting \( \frac{\partial^2 f}{\partial x^2} = 12x, \frac{\partial^2 f}{\partial y^2} = 12x - 18y\) and \( \frac{\partial^2 f}{\partial x \partial y} = 12y \) into formula (15.11) gives
\[ \left( \frac{\partial^2 f}{\partial x^2} \right)^2 - 2 \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 f}{\partial y^2} \right)^2 = 12x(12x - 18y) - (12y)^2 \]
\[ = 144x^2 - 216xy - 144y^2 \quad (\dagger) \]

We test each of the above four stationary points \((5, 0), (-5, 0), (3, 4)\) and \((-3, -4)\) by substituting these \(x\) and \(y\) values into \((\dagger)\). At \((5, 0)\) we have
\[ \left( \frac{\partial^2 f}{\partial x^2} \right)^2 - 2 \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 f}{\partial y^2} \right)^2 = 144x^2 - 216xy - 144y^2 \]
\[ = (144 \times 5^2) - 0 - 0 > 0 \]

Thus \((5, 0)\) is a maximum or minimum. Since \( \frac{\partial^2 f}{\partial x^2} = 12x = 12 \times 5 = 60 > 0 \) therefore we have minimum at \((5, 0)\).

Similarly for \((-5, 0)\) we have maximum or minimum and \( \frac{\partial^2 f}{\partial x^2} = 12x = 12 \times (-5) = -60 < 0 \) therefore this is a maximum.
Testing the point \((3, 4)\) by substituting \(x = 3, \ y = 4\) into (†):

\[
\left(\frac{\partial^2 f}{\partial x^2}\right)\left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = 144x^2 - 216xy - 144y^2
\]

\[
= (144 \times 3^2) - (216 \times 3 \times 4) - (144 \times 4^2)
\]

\[
= 144(9 - 16) - (216 \times 3 \times 4) < 0
\]

Thus \((3, 4)\) is a saddle point. The last stationary point \((-3, -4)\) is also a saddle point because substituting \(x = -3, \ y = -4\) into (†) gives the same result as above.