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CHAPTER 5

Logarithmic, Exponential and Hyperbolic Functions

Additional exercise **5(e)**

15 [*heat transfer*] The temperature distribution, $\theta(x)$, along a fin of length L is given by

$$\theta(x) = Ae^{kx} + Be^{-kx}$$

where A, B and k are constants and x is the distance along the fin. A and B can be evaluated from

$$A + B = \theta_0 \text{ and } Ae^{kL} + Be^{-kL} = \theta_L$$

(θ_0 is the temperature at the base of the fin and θ_L is the temperature at length L). Show that

i $A = \dfrac{\theta_L - \theta_0 e^{-kL}}{e^{kL} - e^{-kL}}$

ii $B = \dfrac{\theta_0 e^{kL} - \theta_L}{e^{kL} - e^{-kL}}$

iii $\theta(x) = \dfrac{\theta_L \sinh(kx) + \theta_0 \sinh[k(L - x)]}{\sinh(k\mathrm{L})}$