CHAPTER 10

Complex Numbers

Additional miscellaneous exercise **10**

22 The transfer function of a simple amplifier is given by

$$A = \frac{A_0}{1 + j\omega T_1}$$

where A_0 is the low frequency gain and T_1 is the amplifier time constant.

i Given that $A_0 = 1000$, $T_1 = 0.5 \times 10^{-3}$ s, find the gain ($= |A|$) and phase ($= \arg(A)$) of A at an angular frequency $\omega = 2 \times 10^3$ rad/s.

ii When negative feedback, β, is applied, the transfer function becomes

$$T = \frac{A_0/(1 + j\omega T_1)}{1 + \dfrac{A_0}{1 + j\omega T_1}\beta(1 + j\omega T_2)}$$

(T = new transfer function, T_2 = feedback time constant)

Show that

$$T = \frac{A_0}{1 + A_0\beta + j\omega(T_1 + A_0\beta T_2)}$$

iii Find the gain and phase of T with $\beta = 0.1$ and $T_2 = 1 \times 10^{-4}$ s at $\omega = 2 \times 10^3$ rad/s.

23 [*acoustics*] The specific impedance, z_s, of a fluid column is given by

$$z_s = z_0\left(\frac{e^{j\theta} + ke^{-j\theta}}{e^{j\theta} - ke^{-j\theta}}\right) \quad \text{where } \theta = \frac{2\pi}{\lambda}x$$

(z_0 is the characteristic impedance, k is a real constant and λ is the wavelength). Determine z_s at

a $x = \lambda/2$ **b** $x = \lambda/4$

24 [*fluid mechanics*] The complex potential function, $w = \phi + j\psi$, for a particular flow is given by

$$w = z^a + \frac{c^2}{z^a}$$

where a and c are non-zero real numbers. Show that the potential function ϕ is

$$\phi = \left(r^a + \frac{c^2}{r^a}\right)\cos(a\theta)$$

and the stream function ψ is

$$\psi = \left(r^a - \frac{c^2}{r^a}\right)\sin(a\theta)$$

where $r = |z|$ and $\theta = \arg(z)$.