15. We are given

$$u = u(x,t) = \left[A\sin(\omega t) + B\cos(\omega t)\right] C\sin\left(\frac{\omega x}{k}\right)$$

Let $f(x) = C\sin\left(\frac{\omega x}{k}\right)$ and $g(t) = A\sin(\omega t) + B\cos(\omega t)$ then we have

$$u = g(t)\, f(x)$$

For $\frac{\partial u}{\partial x}$ we differentiate $f(x)$ whilst treating $g(t)$ as a constant

$$u = g(t) C\sin\left(\frac{\omega x}{k}\right)$$

$$\frac{\partial u}{\partial x} = g(t)\frac{\omega C}{k}\cos\left(\frac{\omega x}{k}\right)$$

Thus

$$\frac{F}{SE} = \frac{\partial u}{\partial x} = \frac{\omega C}{k} g(t)\cos\left(\frac{\omega x}{k}\right) \qquad (*)$$

Next we find $\frac{\partial^2 u}{\partial t^2}$;

$$u = \left[A\sin(\omega t) + B\cos(\omega t)\right] f(x)$$

$$\frac{\partial u}{\partial t} = \left[\omega A\cos(\omega t) - \omega B\sin(\omega t)\right] f(x)$$

$$\frac{\partial^2 u}{\partial t^2} = \left[-\omega^2 A\sin(\omega t) - \omega^2 B\cos(\omega t)\right] f(x)$$

$$= -\omega^2\left[A\sin(\omega t) + B\cos(\omega t)\right] f(x) = -\omega^2 g(t)\, f(x)$$

Substituting this into F gives

$$F = -m\frac{\partial^2 u}{\partial t^2} = m\omega^2 g(t)\, f(x)$$

Rearranging the given equation $\frac{F}{SE} = \frac{\partial u}{\partial x}$ to make SE the subject and substituting gives

$$SE = \frac{F}{\partial u/\partial x}$$

$$= \frac{m\omega^2 g(t)\, f(x)}{\frac{\omega C}{k} g(t)\cos\left(\frac{\omega x}{k}\right)}$$

$$= \frac{km\omega C\sin\left(\frac{\omega x}{k}\right)}{C\cos\left(\frac{\omega x}{k}\right)} = km\omega\tan\left(\frac{\omega x}{k}\right)$$