22. (i) We have $\omega T_1 = 2\times 10^3 \times 0.5\times 10^{-3} = 1$

$$A = \frac{1000}{1+j} = \frac{1000\angle 0°}{\sqrt{2}\angle 45°} \underbrace{=}_{\text{using (10.18)}} \frac{1000}{\sqrt{2}}\angle(-45°)$$

Gain, $|A| = \frac{1000}{\sqrt{2}} = 707.11$ and, phase $\arg(A) = -45°$

(ii) Multiplying the numerator and denominator of T by $1 + j\omega T_1$ gives

$$T = \frac{A_0}{1 + j\omega T_1 + A_0\beta + j\omega T_2 A_0\beta} = \frac{A_0}{1 + A_0\beta + j\omega(T_1 + T_2 A_0\beta)}$$

(iii) Substituting $\omega T_1 = 1$, $\omega T_2 = 2\times 10^3 \times 1\times 10^{-4} = 0.2$ and $A_0\beta = 1000\times 0.1 = 100$ into T gives

$$T = \frac{1000}{(1+100) + j(1+(0.2\times 100))}$$

$$= \frac{1000}{101 + j21} = \frac{1000\angle 0°}{103.16\angle 11.75°} = 9.69\angle(-11.75°)$$

Gain $= 9.69$, Phase $= -11.75°$

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23. (a) Putting $x = \frac{\lambda}{2}$ into $\theta = \frac{2\pi}{\lambda}x$ gives

$$\theta = \frac{2\pi}{\lambda}\frac{\lambda}{2} = \pi \quad \text{[Cancelling]}$$

By (10.25) we have $e^{j\theta} = e^{j\pi} = [\cos(\pi) + j\sin(\pi)] = -1$ also $e^{-j\pi} = -1$. Thus

$$z_s = z_0\left(\frac{-1-k}{-1+k}\right)$$

$$= z_0\left(\frac{-(1+k)}{-(1-k)}\right) = z_0\left(\frac{1+k}{1-k}\right) \qquad \text{[Cancelling the negative signs]}$$

(b) $\theta = \frac{2\pi}{\lambda}\frac{\lambda}{4} = \frac{\pi}{2}$. By (10.25)

$$e^{j\theta} = e^{j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right) = j$$

and $e^{-j\theta} = e^{-j\pi/2} = -j$. We have

$$z_s = z_0\left(\frac{j - jk}{j + jk}\right) \underbrace{=}_{\text{cancelling the } j\text{'s}} z_0\left(\frac{1-k}{1+k}\right)$$

(10.18) $\quad \frac{r\angle A}{q\angle B} = \frac{r}{q}\angle(A - B)$

(10.13) $\quad \frac{a + jb}{c + jd} = \frac{(a + jb)(c - jd)}{c^2 + d^2}$

(10.25) $\quad e^{j\theta} = \cos(\theta) + j\sin(\theta)$

24. Let $z = re^{j\theta}$ which gives

$$z^a = r^a e^{ja\theta}$$

Using this we have

$$w = ø + j\psi = r^a e^{ja\theta} + \frac{c^2}{r^a} e^{-ja\theta}$$

$$= r^a \underbrace{\left(\cos(a\theta) + j\sin(a\theta)\right)}_{\text{by (10.25)}} + \frac{c^2}{r^a} \underbrace{\left(\cos(a\theta) - j\sin(a\theta)\right)}_{\text{by (10.26)}}$$

Equating real and imaginary parts

$$\phi = r^a \cos(a\theta) + \frac{c^2}{r^a}\cos(a\theta) = \left(r^a + \frac{c^2}{r^a} \right)\cos(a\theta)$$

$$\psi = r^a \sin(a\theta) - \frac{c^2}{r^a}\sin(a\theta) = \left(r^a - \frac{c^2}{r^a} \right)\sin(a\theta)$$

(10.25) $\quad e^{j\theta} = \cos(\theta) + j\sin(\theta)$

(10.26) $\quad e^{-j\theta} = \cos(\theta) - j\sin(\theta)$