Additional material

Chapter 2

SECTION G **Applications of equations to electrical circuits**

By the end of this section you will be able to:

- state Kirchhoff's laws
- apply Kirchhoff's laws to electrical circuits
- solve simultaneous equations resulting from Kirchhoff's laws
- solve simultaneous equations using a computer algebra system

G1 **Modelling electrical circuits**

If you are undertaking an electrical-related discipline then you may have covered Kirchhoff's laws in an electrical principles module. However, if you have not covered these laws then 2.7 and 2.8 give Kirchhoff's current and voltage laws respectively.

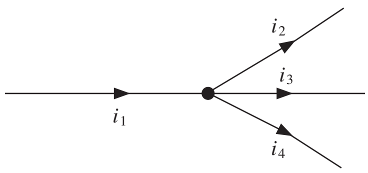
Kirchhoff's current law states

2.7 current entering a node = current leaving a node

For Fig. 43:

$$i_1 = i_2 + i_3 + i_4$$

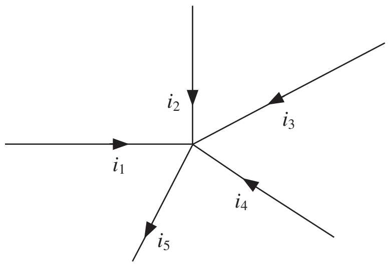
Fig. 43



Example 20

Obtain equations relating the currents i_1, i_2, i_3, i_4 and i_5 of Fig. 44.

Fig. 44



Solution

All the currents apart from i_5 are entering the node, therefore

$$i_5 = i_1 + i_2 + i_3 + i_4$$

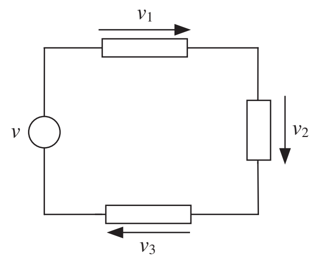
Kirchhoff's voltage law states

2.8 applied voltage = sum of the voltage drops across each component in a loop

For the circuit of Fig. 45 we have

$$v = v_1 + v_2 + v_3$$

Fig. 45

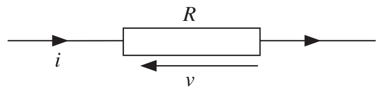


Ohm's law states that:

2.9 $v = iR$

where i is the current flowing through the resistor R and v is the voltage across the resistor R (Fig. 46).

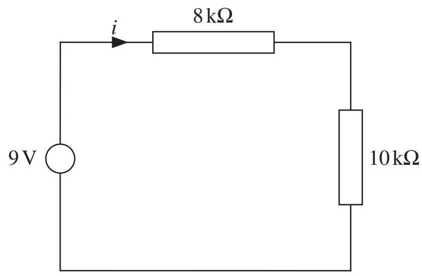
Fig. 46



Example 21

Find the current i flowing through the circuit of Fig. 47.

Fig. 47



Solution

Remember 8 kΩ is 8 kiloΩ and equals 8×10^3 Ω. By applying Kirchhoff's voltage law **2.8** to the circuit of Fig. 47 we have

$$9 = (\text{voltage drop across } 8\text{ k}\Omega) + (\text{voltage drop across } 10\text{ k}\Omega)$$

$$= i(8\text{k}) + i(10\text{k}) \qquad \text{[by applying Ohm's law]}$$

$$= i(18\text{k})$$

$$9 = (18 \times 10^3)i$$

Therefore

$$i = \frac{9}{18 \times 10^3} = 5 \times 10^{-4}$$

$$i = 0.5 \times 10^{-3} = 0.5 \text{ mA}$$

The unit mA is milliamps, 10^{-3} A.

Consider the case where we have more than one loop in the circuit. A direction (normally clockwise) for the current is chosen. The current is positive when in this direction (clockwise) and negative when in the opposite direction (anticlockwise).

Example 22

Determine the currents i_1 and i_2 in the circuit of Fig. 48.

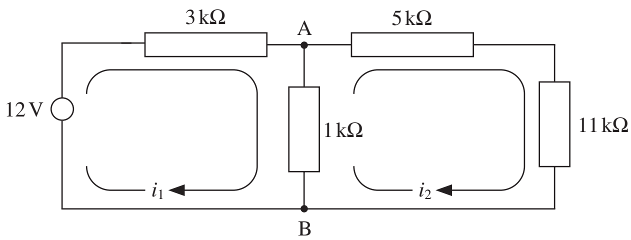


Fig. 48

Solution

What current is flowing from A to B?

$$i_1 - i_2$$

because i_1 and i_2 are going in the opposite directions. Consider the two loops. By applying Kirchhoff's voltage law 2.8 to the first loop, we have

$$12 = (\text{voltage drop across 3 k}\Omega) + (\text{voltage drop across 1 k}\Omega)$$

$$\underbrace{=}_{\text{by 2.9}} (3\text{k})i_1 + (1\text{k})(i_1 - i_2)$$

$$= (3\text{k})i_1 + (1\text{k})i_1 - (1\text{k})i_2$$

$$= (4\text{k})i_1 - (1\text{k})i_2$$

† $$12 = (4 \times 10^3)i_1 - (1 \times 10^3)i_2$$

Applying 2.8 to the second loop:

$$\underbrace{0}_{\substack{\text{no voltage source in}\\\text{the second loop}}} = (\text{voltage drop across 5 k}\Omega) + (\text{voltage drop across 11 k}\Omega) + (\text{voltage drop across 1 k}\Omega)$$

$$0 = (5\text{k})i_2 + (11\text{k})i_2 + 1\text{k}(\underbrace{i_2 - i_1}_{\text{current from B to A}})$$

$$= (5\text{k} + 11\text{k} + 1\text{k})i_2 - (1\text{k})i_1$$

$$= (17\text{k})i_2 - (1\text{k})i_1$$

$$0 = (17 \times 10^3)i_2 - (1 \times 10^3)i_1$$

This gives

$$(1 \times 10^3)i_1 = (17 \times 10^3)i_2$$

†† $$i_1 = 17i_2 \qquad [\text{Cancelling } 10^3\text{'s}]$$

2.8 applied voltage = sum of voltage drops across each component in a loop

2.9 $v = iR$

Example 22 *continued*

We need to solve the simultaneous equations obtained:

† $$12 = (4 \times 10^3)i_1 - (1 \times 10^3)i_2$$

†† $$i_1 = 17i_2$$

We can substitute $i_1 = 17i_2$ into †:

$$12 = (4 \times 10^3)17i_2 - (1 \times 10^3)i_2$$
$$= (67 \times 10^3)i_2$$

Hence

$$i_2 = \frac{12}{67 \times 10^3} = 1.79 \times 10^{-4} \text{ A}$$

? **How can we find i_1?**

Substitute $i_2 = 1.79 \times 10^{-4}$ into $i_1 = 17i_2$:

$$i_1 = 17 \times (1.79 \times 10^{-4})$$
$$= 3.04 \times 10^{-3} \text{ A}$$

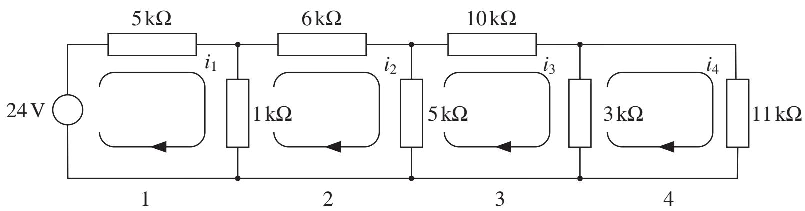
We have $i_1 = 3.04$ mA and $i_2 = 0.18$ mA.

If we consider more than two loops in a circuit then we can set up the equations using Kirchhoff's and Ohm's laws as above. However as for solving these equations, it is easier to use modern technology since it eradicates the drudgery out of the calculations. In the example below we have used a computer algebra system (MAPLE). It might be more convenient to use a graphical calculator because of its portability.

Example 23

Obtain the values of the currents i_1, i_2, i_3 and i_4 in the circuit of Fig. 49.

Fig. 49



Solution

Considering each loop separately.

Example 23 *continued*

Loop 1: Applying 2.8 we have

$$24 = (\text{voltage drop across } 5\text{ k}\Omega) + (\text{voltage drop across } 1\text{ k}\Omega)$$
$$= (5\text{k})i_1 + (1\text{k})(i_1 - i_2) \qquad [\text{by } 2.9\ ]$$
$$= (6\text{k})i_1 - (1\text{k})i_2$$
$$24 = (6 \times 10^3)i_1 - (1 \times 10^3)i_2$$

Loop 2: Similarly

$$0 = (\text{voltage drop across } 6\text{ k}\Omega) + (\text{voltage drop across } 5\text{ k}\Omega) + (\text{voltage drop across } 1\text{ k}\Omega)$$
$$= (6\text{k})i_2 + (5\text{k})(i_2 - i_3) + (1\text{k})(i_2 - i_1)$$
$$= (12\text{k})i_2 - (1\text{k})i_1 - (5\text{k})i_3$$
$$0 = -(1 \times 10^3)i_1 + (12 \times 10^3)i_2 - (5 \times 10^3)i_3$$

Loop 3: We have

$$0 = (\text{voltage drop across } 10\text{ k}\Omega) + (\text{voltage drop across } 3\text{ k}\Omega) + (\text{voltage drop across } 5\text{ k}\Omega)$$
$$= (10\text{k})i_3 + (3\text{k})(i_3 - i_4) + (5\text{k})(i_3 - i_2)$$
$$= (18\text{k})i_3 - (3\text{k})i_4 - (5\text{k})i_2$$
$$0 = -(5 \times 10^3)i_2 + (18 \times 10^3)i_3 - (3 \times 10^3)i_4$$

Loop 4:

$$0 = (\text{voltage drop across } 11\text{ k}\Omega) + (\text{voltage drop across } 3\text{ k}\Omega)$$
$$= (11\text{k})i_4 + (3\text{k})(i_4 - i_3)$$
$$= (14\text{k})i_4 - (3\text{k})i_3$$
$$0 = -(3 \times 10^3)i_3 + (14 \times 10^3)i_4$$

Combining these four equations gives

$$(6 \times 10^3)i_1 - (1 \times 10^3)i_2 = 24$$
$$-(1 \times 10^3)i_1 + (12 \times 10^3)i_2 - (5 \times 10^3)i_3 = 0$$
$$-(5 \times 10^3)i_2 + (18 \times 10^3)i_3 - (3 \times 10^3)i_4 = 0$$
$$-(3 \times 10^3)i_3 + (14 \times 10^3)i_4 = 0$$

2.8 applied voltage = sum of voltage drops across each component in a loop
2.9 $v = iR$

Example 23 *continued*

Solving these using MAPLE we obtain (*10^3 can also be replaced by e3 in MAPLE)

```
> eqn1:=(6*10^3)*i[1] - (1*10^3)*i[2] = 24;
```

$$\text{eqn 1} := 6000\, i_1 - 1000\, i_2 = 24$$

```
> eqn2:= - (1*10^3)*i[1] + (12*10^3)*i[2] - (5*10^3)*i[3] = 0;
```

$$\text{eqn 2} := -1000\, i_1 + 12000\, i_2 - 5000\, i_3 = 0$$

```
> eqn3:= - (5*10^3)*i[2] + (18*10^3)*i[3] - (3*10^3)*i[4] = 0;
```

$$\text{eqn 3} := -5000\, i_2 + 18000\, i_3 - 3000\, i_4 = 0$$

```
> eqn4:= - (3*10^3)*i[3] + (14*10^3)*i[4] = 0;
```

$$\text{eqn 4} := -3000\, i_3 + 14000\, i_4 = 0$$

```
> evalf (solve ({eqn1, eqn2, eqn3, eqn4}));
```

$$\{i_1 = .004064145714,\ i_3 = .0001108691348,\ i_4 = .00002375767175,\ i_2 = .0003848742823\}$$

Rounding to 2 d.p. we have

$$i_1 = 4.06 \text{ mA},\ i_2 = 0.38 \text{ mA},\ i_3 = 0.11 \text{ mA and } i_4 = 0.02 \text{ mA}$$

SUMMARY

Kirchhoff's current law:

2.7 current entering a node = current leaving a node

Kirchoff's voltage law:

2.8 applied voltage = sum of voltage drops across each component in a loop

Ohm's law:

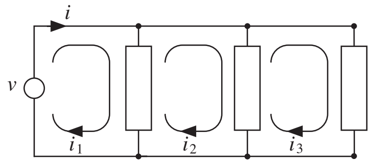
2.9 $v = iR$

Exercise 2(g)

Solutions are given at the end of this additional material.
Complete solutions are in this website.

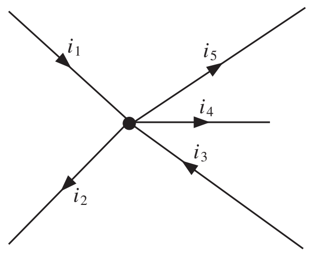
1 Ascertain an expression relating the currents i, i_1, i_2 and i_3 for the circuit of Fig. 50.

Fig. 50



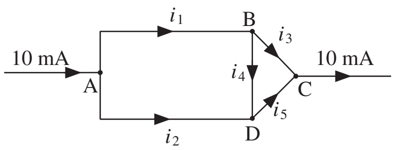
2 Write an expression relating the currents i_1, i_2, i_3, i_4 and i_5 for the circuit of Fig. 51.

Fig. 51



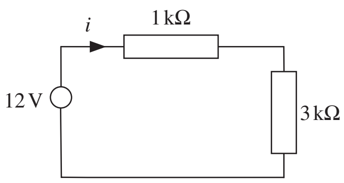
3 Obtain four relationships, one for each node A, B, C, D, between the currents i_1, i_2, i_3, i_4 and i_5 for the circuit of Fig. 52.

Fig. 52



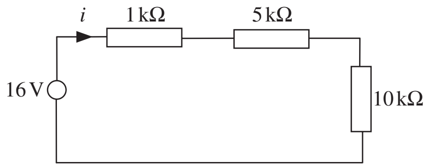
4 Find the current i flowing through the circuit of Fig. 53.

Fig. 53



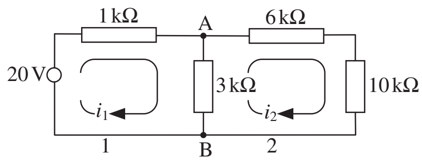
5 Obtain a value for the current i of the circuit of Fig. 54.

Fig. 54



6 Obtain the values of the currents i_1 and i_2 for the circuit of Fig. 55.

Fig. 55



7 Find the currents i_1, i_2 and i_3 shown in the circuit of Fig. 56.

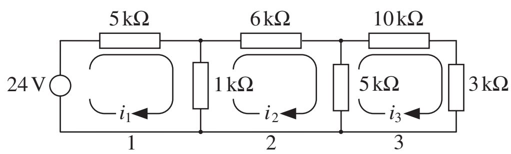


Fig. 56

8 Establish the values of the currents i_1, i_2 and i_3 for the circuit of Fig. 57.

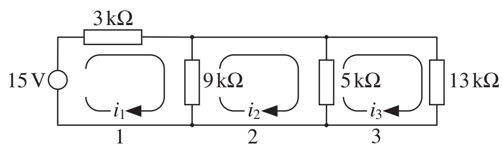


Fig. 57

9 Find the values of the currents i_1, i_2, i_3 and i_4 for the circuit of Fig. 58.

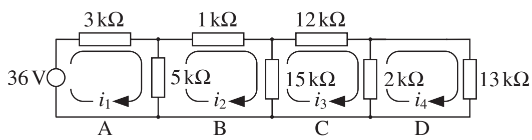


Fig. 58

10 Determine the values of currents i_1, i_2 and i_3 for the circuit of Fig. 59.

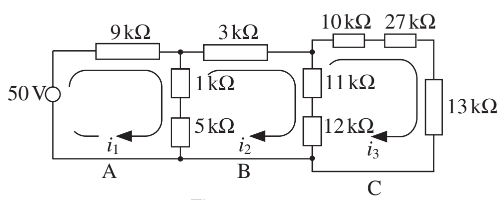


Fig. 59