Chapter 3

G Step function

In electronics engineering, one of the most important functions is the step function, denoted $H(t)$, which is defined as

3.8 $$H(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

$H(t)$ is a function that depends on time, t, and has a value of zero for $t < 0$ and one for $t \geq 0$.

What does the graph of this function look like?

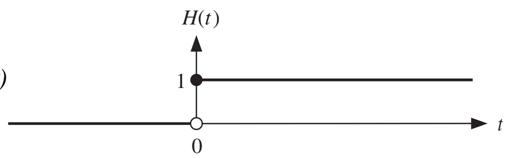
The graph of $H(t)$ is shown in Fig. 28.

The function jumps at $t = 0$ and has a value of 1 at this point and for $t > 0$.

$H(t)$ is sometimes called the 'switch' function (it switches on at $t = 0$).

Why is the step function denoted by $H(t)$?
It was **Oliver Heaviside** (1850–1925) who developed these step functions, hence $H(t)$. He was born in Camden Town, London and at a young age became deaf. However he was interested in academic subjects but detested the rigour of mathematics and chose to publish papers in electromagnetism. In 1891 he was elected a Fellow of the prestigious Royal Society. So the H in the step function refers to Oliver Heaviside. Step function is also known as 'Heaviside' function.

Fig. 28
Graph of $H(t)$



What do ● and ○ signify in the graph of Fig. 28?

The points ● and ○ at $t = 0$ represent the fact that $H(t)$ has a value of 1 at this point and **not** zero.

There is also the delayed step function, $H(t - a)$, given by

3.9 $$H(t-a) = \begin{cases} 1 & \text{if } t \geq a \\ 0 & \text{if } t < a \end{cases}$$

This $H(t - a)$ switches on at $t = a$. The graph has the shape shown in Fig. 29.

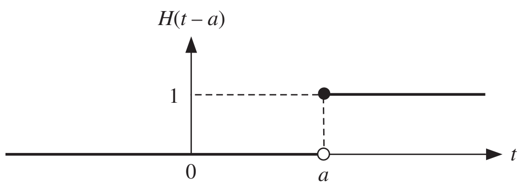


Fig. 29 Graph of $H(t - a)$

Notice that the graph jumps at $t = a$ to a value of 1 for the delayed step function.

Step functions may have other values besides 1. For example, the graph of $bH(t - a)$ has the shape shown in Fig. 30.

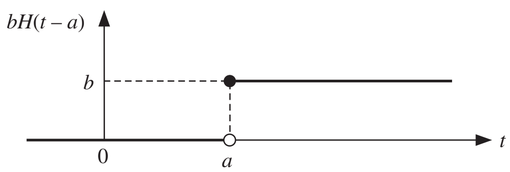


Fig. 30 Graph of $bH(t - a)$

The graph hops from zero to a value of b at a and then stays at this value. It is defined by

3.10 $$bH(t-a) = \begin{cases} b & \text{if } t \geq a \\ 0 & \text{if } t < a \end{cases}$$

Let's try some examples. From now on we will not always plot graphs with ● and ○ and will assume that the graph follows the definition given above in 3.8, 3.9 and 3.10.

Example 25 *electronics*

The voltage, $v(t)$, applied to a circuit is given by

a $v(t) = H(t-3)$ **b** $v(t) = 5H(t-3)$ **c** $v(t) = 5H(t-1) - 5H(t-2)$

Sketch these functions on different axes.

Solution

a The graph of $v(t) = H(t-3)$ means the graph switches on at $t = 3$ and has a value of 1. More rigorously, we can use 3.9 with $a = 3$ which gives

$$H(t-3) = \begin{cases} 1 & \text{if } t \geq 3 \\ 0 & \text{if } t < 3 \end{cases}$$

and the graph has the shape shown in Fig. 31.

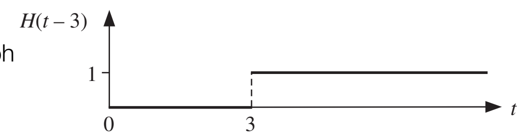


Fig. 31 Graph of $H(t-3)$

b The graph $v(t) = 5H(t-3)$ switches on at $t = 3$ and has a value of 5. Putting $b = 5$ and $a = 3$ into 3.10 gives

$$5H(t-3) = \begin{cases} 5 & \text{if } t \geq 3 \\ 0 & \text{if } t < 3 \end{cases}$$

(see Fig. 32).

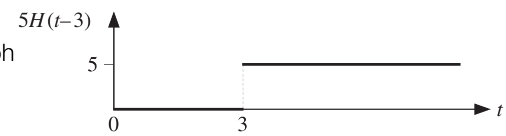


Fig. 32 Graph of $5H(t-3)$

c For $v(t) = 5H(t-1) - 5H(t-2)$, we can consider each part by using 3.10:

$$5H(t-1) = \begin{cases} 5 & \text{if } t \geq 1 \\ 0 & \text{if } t < 1 \end{cases} \quad \text{and} \quad 5H(t-2) = \begin{cases} 5 & \text{if } t \geq 2 \\ 0 & \text{if } t < 2 \end{cases}$$

For $t < 1$, $5H(t-1) = 0$ and $5H(t-2) = 0$, so $v(t) = ?$

$$v(t) = 0 - 0 = 0$$

For $1 \leq t < 2$, $5H(t-1) = 5$ and $5H(t-2) = 0$, so $v(t) = ?$

$$v(t) = 5 - 0 = 5$$

For $t \geq 2$, $5H(t-1) = 5$ and $5H(t-2) = 5$, so $v(t) = 5 - 5 = 0$.

3.10 $$bH(t-a) = \begin{cases} b & \text{if } t \geq a \\ 0 & \text{if } t < a \end{cases}$$

Example 25 *continued*

Combining these three pieces we have the graph shown in Fig. 33.

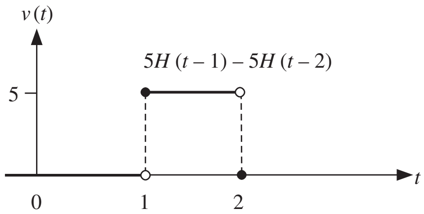


Fig. 33 Graph of $5H(t-1) - 5H(t-2)$

Observe that the graph of $v(t) = 5H(t-1) - 5H(t-2)$ is a pulse of value 5 between 1 and 2. Also at $t = 1$, $v(t) = 5$ and at $t = 2$, $v(t) = 0$. A function of this format, $5H(t-1) - 5H(t-2)$, will always be a pulse. For example, the general function $f(t) = H(t-a) - H(t-b)$ has a pulse of height 1 between $t = a$ and $t = b$, and zero elsewhere.

Step functions can also take up other values besides constants. For example, the graph of $g(t)H(t-a)$ has the shape shown in Fig. 34.

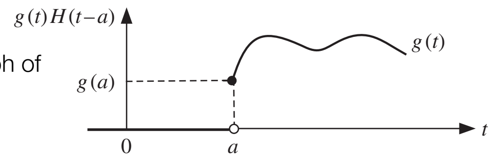


Fig. 34 Graph of $g(t)H(t-a)$

The graph hops from zero to a value of $g(a)$ at a and then traces the graph of $g(t)$ for $t \geq a$. It is defined by

3.11 $$g(t)H(t-a) = \begin{cases} g(t) & \text{if } t \geq a \\ 0 & \text{if } t < a \end{cases}$$

Example 26 *electronics*

The input voltage, $v(t)$, to an amplifier is given by

$$v(t) = t^2 H(t-2)$$

Sketch this function.

Solution

For $v(t) = t^2 H(t-2)$, putting $a = 2$ and $g(t) = t^2$ into

3.11 $$g(t)H(t-a) = \begin{cases} g(t) & \text{if } t \geq a \\ 0 & \text{if } t < a \end{cases}$$

Example 26 *continued*

gives

$$t^2H(t-2) = \begin{cases} t^2 & \text{if } t \geq 2 \\ 0 & \text{if } t < 2 \end{cases}$$

How do we sketch this graph?

Well, $v(t)$ switches on at $t = 2$ and then it traces the graph of t^2 from 2 onwards (Fig. 35).

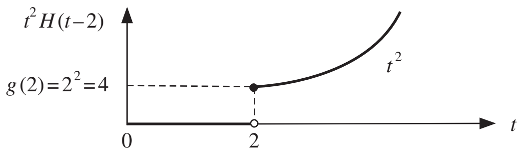


Fig. 35 Graph of $t^2H(t-2)$

SUMMARY

The basic step function is defined by

3.8 $$H(t) = \begin{cases} 1 & \text{if } t \geq 0 \\ 0 & \text{if } t < 0 \end{cases}$$

The step function is sometimes called the 'switch' function.

Exercise 3(g)

Solutions are given at the end of this additional material. Complete solutions are in this website.

1 Sketch the following functions:

a $f(t) = H(t-2)$

b $f(t) = 2H(t-2)$

c $f(t) = tH(t-1)$

d $f(t) = (t^2 - 2t + 1)H(t-1)$

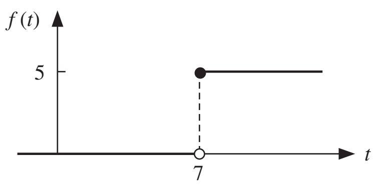
e $f(t) = H(t-2) - H(t-3)$

f $f(t) = t^2[H(t-2) - H(t-3)]$

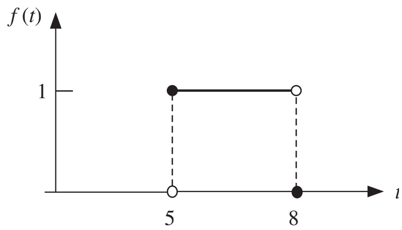
g $f(t) = 5H(t-2) - 6H(t-3)$

2 Write expressions for the functions, $f(t)$, of the following graphs:

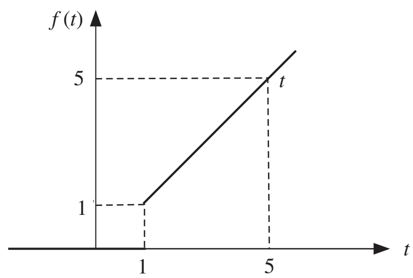
a



b



c



Exercise 3(g)

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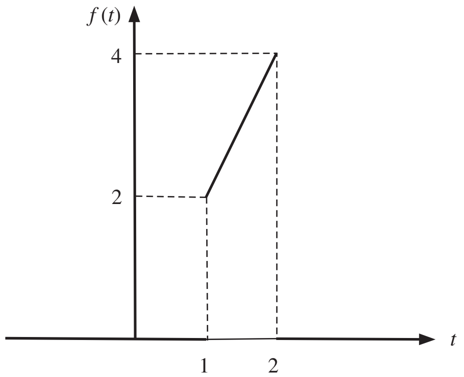
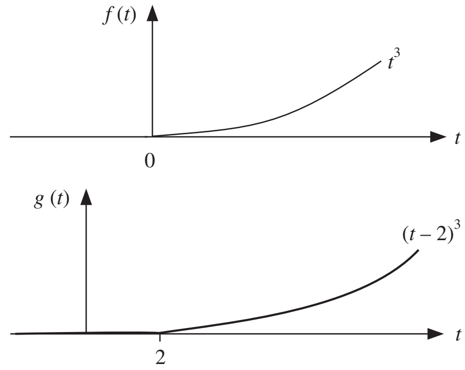
d



3 **i** Write the functions, $f(t)$ and $g(t)$, for the following graphs:



ii Sketch the graph of $f(t) = (t - 5)^3 H(t - 5)$

iii Sketch the graph of $f(t) = (t^3 - 15t^2 + 15t - 125)\, H(t - 5)$

Miscellaneous exercise 3 (extra)

Solutions are given at the end of this additional material.
Complete solutions are in this website.

19 Sketch the following functions:

a $f(t) = H(t - 3)$

b $f(t) = 5H(t - 3) - 5H(t - 4)$

c $f(t) = (t^2 - 4t + 4)H(t - 2)$