1. (a) To find the roots of \( x^2 - 2x - 4 = 0 \) we use the quadratic formula
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\] with \( a = 1 \), \( b = -2 \) and \( c = -4 \):
\[
x = \frac{-(-2) \pm \sqrt{(-2)^2 - [4 \times 1 \times (-4)]}}{2 \times 1}
\]
\[
= \frac{2 \pm \sqrt{4 + 16}}{2}
\]
\[
= \frac{2 \pm \sqrt{20}}{2}
\]
\[
= 1 \pm \sqrt{5} \quad \text{[Because \( \sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5} \)]}
\]
\[
= 1 - \sqrt{5}, 1 + \sqrt{5} = -1.236, 3.236
\]
(b) We can complete the square on \( x^2 - 2x - 4 \) to find where the minimum occurs:
\[
x^2 - 2x - 4 = (x-1)^2 - 1 - 4 = (x-1)^2 - 5
\]
Hence the minimum occurs at \( x = 1 \) with a \( y \) value of \(-5\):

![Graph of \( y = x^2 - 2x - 4 \)]

2. (a) To find the roots of \( 3x^2 - 2x - 9 = 0 \) we use the formula
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\] with \( a = 3 \), \( b = -2 \) and \( c = -9 \):
\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(2) \pm \sqrt{(-2)^2 - [4 \times 3 \times (-9)]}}{2 \times 3} \]

\[ = \frac{2 \pm \sqrt{4+108}}{6} \]

\[ = \frac{2 \pm \sqrt{112}}{6} \]

\[ = \frac{2 \pm 10.583}{6} \]

\[ = \frac{2-10.583}{6}, \quad \frac{2+10.583}{6} = -1.43, \ 2.10 \]

(b) Completing the square on the given quadratic \(3x^2 - 2x - 9\) is more difficult because of the coefficient of \(x^2\) is 3 which means we need to divide by 3. However it is easier to see that the minimum value occurs halfway between the roots, \(-1.43\) and \(2.10\), which is

\[ \frac{-1.43+2.10}{2} = 0.335 \]

3. The expansion of \((1+x)^6\) was carried out in question 1a Exercise 2f:

\[ (1+x)^6 = 1+6x+15x^2+20x^3+15x^4+6x^5+x^6 \]

*How do we find an approximation to \((11/10)^6\)?*

Use the above result of the expansion of \((1+x)^6\) with \(x = \frac{1}{10} = 0.1\) because
\[ \frac{11}{10} = 1 + \frac{1}{10} = 1 + 0.1 \]

To give the final answer correct to three decimal places we need to work to four decimal places:

\[
(1 + 0.1)^6 = 1 + 6(0.1) + 15(0.1)^2 + 20(0.1)^3 + 15(0.1)^4 + 6(0.1)^5 + (0.1)^6
\]

\[= 1 + 0.6 + 0.15 + 0.02 + 0.0015 + 0 + 0\]

\[= 1.7715 = 1.772 \text{ (3 dp)}\]

4. By using Pascal’s triangle for \( n = 5 \) we have 1, 5, 10, 5 and 1. We have

\[(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5 \quad (\ast)\]

*How do we determine \( \left(x^2 + \frac{1}{x}\right)^5 \)?*

Substitute \( a = x^2 \) and \( b = \frac{1}{x} \) into the above \((\ast)\):

\[
\left(x^2 + \frac{1}{x}\right)^5 = \left(x^2\right)^5 + 5\left(x^2\right)^4\left(\frac{1}{x}\right) + 10\left(x^2\right)^3\left(\frac{1}{x}\right)^2 + 10\left(x^2\right)^2\left(\frac{1}{x}\right)^3 + 5x^2\left(\frac{1}{x}\right)^4 + \left(\frac{1}{x}\right)^5
\]

Using the rules of indices \(a^m \cdot a^n = a^{m+n}\)

\[
\equiv x^{10} + 5x^8\left(\frac{1}{x}\right) + 10x^6\left(\frac{1}{x^2}\right) + 10x^4\left(\frac{1}{x^3}\right) + 5x^2\left(\frac{1}{x^4}\right) + \frac{1}{x^5}
\]

Using the rules of indices \(a^m / a^n = a^{m-n}\)

Hence we have \( \left(x^2 + \frac{1}{x}\right)^5 = x^{10} + 5x^7 + 10x^4 + 10x + \frac{5}{x^2} + \frac{1}{x^5} \).

5. *How do we complete the square on \( 3x^2 - 6x + 1 \)?*

Take the coefficient of \( x^2 \), which is 3, out:

\[3x^2 - 6x + 1 = 3\left(x^2 - 2x + \frac{1}{3}\right)\]

\[= 3\left((x - 1)^2 - 1 + \frac{1}{3}\right) = 3\left((x - 1)^2 - \frac{2}{3}\right) = 0\]

*How do we solve \( 3\left((x - 1)^2 - \frac{2}{3}\right) = 0 \)?*

Divide both sides by 3 and then transpose to make \( x \) the subject of the formula:

\[(x - 1)^2 - \frac{2}{3} = 0\]

\[(x - 1)^2 = \frac{2}{3}\]

taking the square root gives \( x - 1 = \pm \sqrt{\frac{2}{3}} \)

Therefore \( x = 1 \pm \sqrt{\frac{2}{3}} = 1 - \frac{2}{\sqrt{3}}, 1 + \frac{2}{\sqrt{3}} \). Using our calculator gives

\[x = 0.184, 1.817\]
6. (a) We need to factorize \( f(x) = 9x^2 - 12x - 5 \) and equate to zero.

\[
9x^2 - 12x - 5 = (3x+1)(3x-5) = 0
\]

Solving this \((3x+1)(3x-5) = 0\) gives \(3x+1 = 0\), \(3x-5 = 0\) which means that \(x = -\frac{1}{3}, \ x = \frac{5}{3}\).

The \(x\)-intercepts are \((-\frac{1}{3}, 0)\) and \((\frac{5}{3}, 0)\).

(b) How do we complete the square on the given quadratic expression \(9x^2 - 12x - 5\)?

Take out 9:

\[
9x^2 - 12x - 5 = 9\left(x^2 - \frac{12}{9}x - \frac{5}{9}\right)
\]

\[
= 9\left(x^2 - \frac{4}{3}x - \frac{5}{9}\right) \quad \text{[Because } \frac{12}{9} = \frac{4}{3} \text{]}
\]

\[
= 9\left[(x - \frac{2}{3})^2 - \frac{4}{9} - \frac{5}{9}\right] \quad \text{[Because } \frac{2}{3} \text{ is half of } \frac{4}{3} \text{]}
\]

\[
= 9\left[(x - \frac{2}{3})^2 - 1\right] = 9\left(x - \frac{2}{3}\right)^2 - 9
\]

Since \(x^2\) coefficient is positive (9) therefore we have a minimum. The minimum value is \(-9\) when \(x = \frac{2}{3}\). Hence the vertex of the graph is \((\frac{2}{3}, -9)\).

7. (a) How do we find the quadratic equation of the given graph?

This means that the maximum value is 3 at \(x = 2\). Since we have maximum therefore the \(x^2\) coefficient is negative. This means we have \(-(x-2)^2\) because the maximum value occurs at \(x = 2\). Hence we have the quadratic \(3 - (x-2)^2\) because the maximum value is 3.

Expanding this out yields
\[3 - (x - 2)^2 = 3 - (x^2 - 4x + 4)\]
\[= 3 - x^2 + 4x - 4 = -1 + 4x - x^2\]

(b) Similarly for the given quadratic:

This time we have a minimum which means that the coefficient of \(x^2\) is positive. Since the minimum occurs at \((-2, -1)\) therefore the quadratic is given by:

\[(x + 2)^2 - 1 = x^2 + 4x + 4 - 1\]
\[= x^2 + 4x + 3\]