Complete solutions to Intro(k)

1. (a) What do 10 and 100 have in common?
A common factor of 10. Divide both by 10:
\[
\begin{align*}
100 : 10 &= 10 \\
10 : 10 &= 1
\end{align*}
\]
We have 100:10 = 10:1.

(b) 23 is a common factor of 69 and 23. Divide both by 23.
\[
\begin{align*}
69 : 23 &= 3 \\
23 : 23 &= 1
\end{align*}
\]
Hence 69:23 = 3:1.

(c) What is a common factor of 5, 25 and 45?
Obviously 5, dividing all three numbers by 5:
\[
\begin{align*}
5 : 5 &= 1 \\
25 : 5 &= 5 \\
45 : 5 &= 9
\end{align*}
\]
Hence 5:25:45 = 1:5:9.

(d) What do 6, 42, 54 and 72 have in common?
A factor of 6. Dividing each by 6:
\[
\begin{align*}
6 : 6 &= 1 \\
42 : 6 &= 7 \\
54 : 6 &= 9 \\
72 : 6 &= 12
\end{align*}
\]

(e) It is easier to use whole numbers rather than fractions. What is the lowest common multiple of 3 and 6?
6, multiply both fractions by 6:
\[
\frac{2}{3} \times 6 = 4, \quad \frac{5}{6} \times 6 = 5
\]
Hence \( \frac{2}{3} : \frac{5}{6} = 4:5 \).

2. (a) We can write ratio as a division. We have
\[
\frac{1\frac{1}{5}}{2\frac{1}{4}} = \frac{1\frac{1}{5}}{2\frac{1}{4}}
\]
\[
1\frac{1}{5} : 2\frac{1}{4} \quad \text{on a calculator gives } \frac{8}{15}, \quad \text{hence }
\]
\[
\frac{1\frac{1}{5}}{2\frac{1}{4}} = 8:15
\]

(b) Similarly evaluating \( 3\frac{1}{2} : 2\frac{5}{12} \) on a calculator gives \( \frac{13}{29} \) which we can write as a top-heavy fraction:
\[
\frac{13}{29} = \frac{(1 \times 29) + 13}{29} = \frac{42}{29}
\]
Hence \( 3\frac{1}{2} : 2\frac{5}{12} = 42:29 \).

3. (a) Multiplying 0.4 and 0.5 by 10:
\[
0.4 \times 10 = 4 \quad \text{and} \quad 0.5 \times 10 = 5
\]
0.4:0.5 = 4:5.

(b) How do we convert 0.52 and 0.72 into whole numbers?
Multiply by 100:
\[
0.52 \times 100 = 52 \\
0.72 \times 100 = 72
\]
Thus 0.52:0.72 = 52:72. What do 52 and 72 have in common?
A factor of 4, dividing by 4:
52 ÷ 4 = 13 and 72 ÷ 4 = 18
We have 52:72 = 13:18 so
0.52:0.72 = 13:18
(c) We need to write $\sqrt{8}$ in terms of $\sqrt{2}$, if possible?

$\sqrt{8} = \sqrt{4 \times 2} = \sqrt{4} \sqrt{2} = 2 \sqrt{2}$

We have $\sqrt{2} : \sqrt{8} = \sqrt{2} : 2 \sqrt{2}$. That is $\sqrt{2}$ and $\sqrt{8}$ have a common factor of $\sqrt{2}$, so dividing both by $\sqrt{2}$:

$\frac{\sqrt{2}}{\sqrt{2}} + \frac{\sqrt{2}}{\sqrt{2}} = 1$

$\frac{\sqrt{8}}{\sqrt{2}} = 2 \sqrt{2} : \sqrt{2} = \frac{2 \sqrt{2}}{\sqrt{2}} = 2$

Therefore $\sqrt{2} : \sqrt{8} = 1 : 2$.

4. What is the total number of parts?

$2 + 3 + 5 = 10$

We divide length 0.64m into 10 equal parts:

$0.64 \div 10 = 0.064m$

Since the ratio is 2:3:5 we multiply 0.064m by 2, 3 and 5 to give the length of each piece:

$2 \times 0.064m = 0.128m$

$3 \times 0.064m = 0.192m$

$5 \times 0.064m = 0.32m$

5. The total number of parts is $8 + 3 = 11$, so the mass of each part is

$66kg \div 11 = 6kg$

Since copper is 8 parts, we multiply 6kg by 8:

$copper mass = 6kg \times 8 = 48kg$

Zinc is 3 parts, so

$zinc mass = 6kg \times 3 = 18kg$

6. What is the total number of parts?

$3 + 4 + 6 = 13$

We divide 45.5kg into 13 equal parts:

$45.5 \div 13 = 3.5$

There is 3.5kg in each part, so

$copper mass = 3 \times 3.5kg = 10.5kg$

$zinc mass = 4 \times 3.5kg = 14kg$

$nickel mass = 6 \times 3.5kg = 21kg$

7. The total number of parts is $2 + 3 + 5 + 4 = 14$. The number of students in each part is evaluated by dividing 1260 by 14:

$1260 \div 14 = 90$

There are 90 students in each part.

Number of students in manufacturing = $2 \times 90 = 180$

Number of students in building services = $3 \times 90 = 270$

Number of students in vehicle = $5 \times 90 = 450$

Number of students in control = $4 \times 90 = 360$