Complete solutions to Exercise 12(a)

1. We have

\[ \theta \]
\[ \begin{array}{c}
12 \text{kN} \\
\theta \\
\text{R} \\
5 \text{kN}
\end{array} \]

By Pythagoras

\[ |R| = \sqrt{12^2 + 5^2} = 13 \text{kN} \]

The angle \( \theta \) is given by

\[ \theta = \tan^{-1} \left( \frac{5}{12} \right) = 22.62^\circ \]

\( \text{R} \) is the force of magnitude 13kN and at an angle of 22.62° below the horizontal.

2. Let \( F_x \) and \( F_y \) be the horizontal and vertical components of \( F \).

\[ \begin{array}{c}
10 \text{kN} \\
30^\circ \\
F_x \\
F_y
\end{array} \]

By (4.5)

\[ F_x = 10 \cos(30^\circ) = 8.66 \text{kN} \text{ horizontal} \]

By (4.4)

\[ F_y = 10 \sin(30^\circ) = 5 \text{kN} \text{ vertical} \]

3. Let \( F_x \) and \( F_y \) be the horizontal and vertical components of \( F \).

\[ \begin{array}{c}
F \\
45^\circ \\
F_x \\
F_y
\end{array} \]

Remember 12000 = 12kN. Applying (4.5) and (4.4) gives

\[ |F_x| = 12 \cos(45^\circ) = 8.49 \text{kN} \]

\[ |F_y| = 12 \sin(45^\circ) = 8.49 \text{kN} \]

The force \( F \) has the components 8.49kN horizontally and 8.49kN vertically.

4. The angle \( \theta \) is given by

\[ \theta = \tan^{-1}(\sqrt{3}) = 66.80^\circ \]

Since \( |F| = 10\text{kN} \) we have

\[ |F_x| = 10 \cos(66.80^\circ) = 3.94 \text{kN} \]

\[ |F_y| = 10 \sin(66.80^\circ) = 9.19 \text{kN} \]

3.94kN horizontally and 9.19kN vertically.

\[(4.4)\quad a = h \sin(\theta)\]

\[(4.5)\quad a = h \cos(\theta)\]
5. We have
\[ \vec{AM} = \vec{AO} + \vec{OM} = -\vec{OA} + \vec{OM} = -\vec{a} + \frac{1}{2}\vec{b} = \frac{1}{2}\vec{b} - \vec{a} \]

6. We have
\[ \vec{MN} = \vec{MC} + \vec{CN} \quad (\dagger) \]

Since M is the midpoint of AC
\[ \vec{MC} = \frac{1}{2}\vec{AC} \]

Similarly \( \vec{CN} = \frac{1}{2}\vec{CB} \). Also \( \vec{CB} = -\vec{BC} \). So
\[ \vec{CN} = -\frac{1}{2}\vec{BC} \]

By observing Fig 26
\[ \vec{AC} = \vec{AB} + \vec{BC} \]

which gives
\[ \vec{MC} = \frac{1}{2}\left(\vec{AB} + \vec{BC}\right) \]

Substituting \( \vec{CN} = -\frac{1}{2}\vec{BC} \) and \( \vec{MC} = \frac{1}{2}\left(\vec{AB} + \vec{BC}\right) \) into \( (\dagger) \) gives
\[ \vec{MN} = \frac{1}{2}\left(\vec{AB} + \vec{BC}\right) - \frac{1}{2}\vec{BC} \]
\[ = \frac{1}{2}\vec{AB} + \frac{1}{2}\vec{BC} - \frac{1}{2}\vec{BC} = \frac{1}{2}\vec{AB} \]

7. Drawing \( \mathbf{F}_1, \mathbf{F}_2 \) and \( \mathbf{F} \) in a triangle we have

\[ \theta = 180^\circ - (60^\circ + 90^\circ) = 30^\circ \]

angle \( C = 30^\circ + 40^\circ = 70^\circ \)

We label \( a \) and \( c \) to the sides opposite the angles A and C respectively.

To find \( |\mathbf{F}| \) we can use the sine rule
\[ \frac{a}{\sin(A)} = \frac{c}{\sin(C)} \]

Substituting \( a = 10, \ A = 60^\circ, \ C = 70^\circ \) into the sine rule gives
\[ \frac{10}{\sin(60^\circ)} = \frac{c}{\sin(70^\circ)} \]
\[ c = \frac{10}{\sin(60^\circ)} \times \sin(70^\circ) = 10.851 \]

\( \mathbf{F} \) is 10.85N horizontally.
8. We have

![Triangle Diagram]

To find the magnitude of \( \mathbf{R} \) we use the sine rule (4.16). Substituting \( c = 12 \), angle \( \angle C = 20^\circ \) and angle \( \angle B = 115^\circ \) into (4.16) gives

\[
\frac{b}{\sin(115^\circ)} = \frac{12}{\sin(20^\circ)}
\]

\[
b = \frac{12}{\sin(20^\circ)} \times \sin(115^\circ) = 31.80
\]

The magnitude of \( \mathbf{R} \) is 31.80 kN

9. We can rotate the forces as follows:

![Diagram of Forces]

By shifting \( \mathbf{F}_2 \) to the end of \( \mathbf{F}_1 \) we have the resultant force, \( \mathbf{R} \), given by:

![Resultant Force Diagram]

We can find the magnitude of \( \mathbf{R} \) by using the cosine rule (4.18). We have \( a = 20 \), \( c = 7 \) and angle \( \angle B = 120^\circ \). Substituting into (4.18) gives

\[
b^2 = a^2 + c^2 - 2ac \cos(B)
\]

\[
b^2 = 20^2 + 7^2 - (2 \times 20 \times 7 \cos(120^\circ)) = 589
\]

\[
b = \sqrt{589} = 24.27
\]

\( |\mathbf{R}| = 24.27 \text{ kN} \). Need to find angle \( \angle A \). We can use the sine rule (4.16).

Substituting \( a = 20 \), \( B = 120^\circ \) and \( b = 24.27 \) into (4.16) gives

\[
\frac{20}{\sin(A)} = \frac{24.27}{\sin(120^\circ)}
\]

\[
\sin(A) = \frac{20 \times \sin(120^\circ)}{24.27} = 0.7137
\]

\[
A = \sin^{-1}(0.7137) = 45.54^\circ
\]

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(4.16) \[ \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)} \]

(4.18) \[ b^2 = a^2 + c^2 - 2ac \cos(B) \]
The resultant force has a magnitude of 24.27\(kN\) and is at an angle of 45.54° from force \(F_1\). Plotting the resultant on the original diagram gives:

Consider the triangle ABC. We can find \(R\) by using the cosine rule on ABC.
The magnitude of \(R\) is represented by \(b\). Substitute \(a = 10\), \(c = 15\) and \(B = 135°\) into

\[
b^2 = a^2 + c^2 - 2ac \cos(B)
\]
gives

\[
b^2 = 10^2 + 15^2 - (2 \times 10 \times 15) \cos(135°) = 537.132
\]

\[
b = \sqrt{537.132} = 23.18
\]

So \(|R| = 23.18kN\). How can we find angle A?
Use sine rule (4.16). Substituting \(a = 10\), \(b = 23.18\) and \(B = 135°\) gives

\[
\frac{10}{\sin(A)} = \frac{23.18}{\sin(135°)}
\]

\[
\sin(A) = \frac{10}{23.18} \times \sin(135°) = 0.305
\]

Hence taking the inverse sin gives

\(A = 17.76°\)

We have