Complete solutions to Exercise 7(i)

1. (a) \( r_1 = -2 \); Let \( f(x) = x^3 - 2x + 7 \), \( f'(x) = 3x^2 - 2 \). So by applying (7.29) we have:

\[
\begin{align*}
  r_2 &= -2 - \frac{f(-2)}{f'(-2)} = -2 - \frac{23}{10} = -2.3 \\
  r_3 &= -2.3 - \frac{f(-2.3)}{f'(-2.3)} = -2.260 \\
  r_4 &= -2.26 - \frac{f(-2.26)}{f'(-2.26)} = -2.258 \quad \text{(2 d.p.)}
\end{align*}
\]

Since \( r_3 = r_4 \), the root of \( x^3 - 2x + 7 = 0 \) is \(-2.26\) (2 d.p.).

(b) \( r_1 = -2 \); Let \( f(x) = x^4 - 3x^3 - 2 \), \( f'(x) = 4x^3 - 6x \). Applying (7.29):

\[
\begin{align*}
  r_2 &= -2 - \frac{f(-2)}{f'(-2)} = -1.90 \\
  r_3 &= -1.9 - \frac{f(-1.9)}{f'(-1.9)} = -1.887 \\
  r_4 &= -1.887 - \frac{f(-1.887)}{f'(-1.887)} = -1.887
\end{align*}
\]

Since \( r_3 = r_4 \) a root of \( x^4 - 3x^3 - 2 = 0 \) is \(-1.89\) correct to two d.p. (Although \(-1.887\) is correct to three d.p.).

(c) \( r_1 = -2 \); Let \( f(x) = e^x - 2x - 5 \), \( f'(x) = e^x - 2 \):

\[
\begin{align*}
  r_2 &= -2 - \frac{f(-2)}{f'(-2)} = -2 - \frac{0.865}{-1.865} = -2.464 \\
  r_3 &= -2.464 - \frac{f(-2.464)}{f'(-2.464)} = -2.464 - \left( \frac{0.013}{-1.915} \right) = -2.457
\end{align*}
\]

\( r_2 = -2.46 \) correct to two d.p. and \( r_3 = -2.46 \) correct to two d.p. To the required accuracy \( r_2 = r_3 \) so a root of \( e^x - 2x - 5 = 0 \) is \(-2.46\) (2 d.p.).

2. Let

\[
\begin{align*}
  f(\lambda) &= \lambda^3 + 28\lambda^2 + 231\lambda + 541 \\
  f'(\lambda) &= 3\lambda^2 + 56\lambda + 231
\end{align*}
\]

For the first root (close to \(-4\)), take \( r_1 = -4 \):

\[
\begin{align*}
  r_2 &\equiv -4 - \frac{f(-4)}{f'(-4)} = -4 - \frac{-4.0182}{-4.0182} = -4.0182 \\
  r_3 &\equiv -4.0182 - \frac{f(-4.0182)}{f'(-4.0182)} = -4.0183
\end{align*}
\]

\(-4.018 = r_2 = r_3\) correct to three d.p., so the root close to \(-4\) is \(-4.018\).

For the 2nd root (close to \(-9\)) take \( r_1 = -9 \):

\[
(7.29) \quad r_{n+1} = r_n - \frac{f(r_n)}{f'(r_n)}
\]
\[ r_2 = -9 - \frac{f(-9)}{f'(-9)} = -8.9667 \]
\[ r_3 = -8.9667 - \frac{f(-8.9667)}{f'(-8.9667)} = -8.9666 - 8.976 \text{ (to 3 d.p.)} \]

\[ r_2 = r_3 = -8.967 \text{ (to three d.p.), so the root close to } -9 \text{ is } -8.967. \]

The third root (close to -15), take \( r_1 = -15 \):
\[ r_2 = -15 - \frac{f(-15)}{f'(-15)} = -15.0151 \]
\[ r_3 = -15.0151 - \frac{f(-15.0151)}{f'(-15.0151)} = -15.0151 \]

\( r_2 = r_3 \), so the root close to -15 is -15.015 correct to three d.p.

All three roots correct to three d.p. are -4.018, -8.967 and -15.015.

3. Take \( r_1 = 10 \) and let
\[ f(v) = v^3 - 6v^2 - 348v + 3112 \quad \text{and} \quad f'(v) = 3v^2 - 12v - 348 \]

By (7.29)
\[ r_2 = 10 - \frac{f(10)}{f'(10)} = 10 - \frac{32}{-168} = 10.1905 \]
\[ r_3 = 10.19 - \frac{f(10.19)}{f'(10.19)} = 10.1960 \]
\[ r_4 = 10.196 - \frac{f(10.196)}{f'(10.196)} = 10.1960 \]

Since \( r_3 = r_4 \), so \( v = 10.196 \text{ m/s} \), correct to three d.p.

4. Let \( f(x) = x^3 - 3x^2 + 2x - 1 \), so we need to solve the equation \( f(x) = 0 \) because \( \frac{WL}{EI} \neq 0 \).

Try some obvious values of \( x \): \( x = 1, 2 \) and \( 3 \)
\[ x = 1; \quad f(1) = 1 - 3 + 2 - 1 = -1 \]
\[ x = 2; \quad f(2) = 2^3 - 3(2)^2 + 2(2) - 1 = -1 \]
\[ x = 3; \quad f(3) = 3^3 - 3(3)^2 + 2(3) - 1 = 5 \]

We know there is a root between \( x = 2 \) and \( x = 3 \) because \( f(x) \) goes from negative to positive. Since -1 is closer to zero, try \( r_1 = 2 \).
\[ f(x) = x^3 - 3x^2 + 2x - 1 \]
\[ f'(x) = 3x^2 - 6x + 2 \]

By repeated use of (7.29) we have

\[ r_{n+1} = r_n - \frac{f(r_n)}{f'(r_n)} \]
\[ r_2 = 2 - \frac{f(2)}{f'(2)} = 2.5 \]
\[ r_3 = 2.5 - \frac{f(2.5)}{f'(2.5)} = 2.348 \]
\[ r_4 = 2.348 - \frac{f(2.348)}{f'(2.348)} = 2.325 \]
\[ r_5 = 2.325 - \frac{f(2.325)}{f'(2.325)} = 2.325 \]

Since \( r_4 = r_5 \), the distance along the beam, where there is zero deflection is \( 2.325 \text{m} \) (3 d.p.).