Complete solutions to Exercise 9(a)

1. We apply the trapezium rule, (9.1), with \(h = 0.5, \ a = 0, \ b = 3\) and \(y_0 = 3.2, \ y_1 = 5.6, \ y_2 = 7.0, \ y_3 = 7.7, \ y_4 = 8.4, \ y_5 = 9.9\) and \(y_6 = 11.6\):

   Impulse of force \(= \int_0^3 F dt \approx \frac{0.5}{2} \left[ 3.2 + 2(5.6 + 7.0 + 7.7 + 8.4 + 9.9) + 11.6 \right]
   \[= 23 \text{ N s} \]

2. (a) We can first establish a table and then slot the values into (9.1):

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e^{-x^2})</td>
<td>1</td>
<td>0.939</td>
<td>0.779</td>
<td>0.570</td>
<td>0.368</td>
</tr>
</tbody>
</table>

Applying (9.1) with uniform width \(h = 0.25\) and the \(y\) values read from the second row:

\[\int e^{-x^2} dx \approx \frac{0.25}{2} \left[ 1 + 2(0.939 + 0.779 + 0.570) + 0.368 \right] = 0.743\]

(b) Since we have 4 equal intervals, so \(h = \frac{\pi/2 - 0}{4} = \frac{\pi}{8}\). As (a) we form a table of values for \(\sqrt{\cos(x)}\):

<table>
<thead>
<tr>
<th>(x)</th>
<th>(\pi/8)</th>
<th>(\pi/4)</th>
<th>(3\pi/8)</th>
<th>(\pi/2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sqrt{\cos(x)})</td>
<td>1</td>
<td>0.961</td>
<td>0.841</td>
<td>0.619</td>
</tr>
</tbody>
</table>

Using (9.1):

\[\int_0^{\pi/2} \sqrt{\cos(x)} dx \approx \frac{\pi/8}{2} \left[ 1 + 2(0.961 + 0.841 + 0.619) + 0 \right] = 1.147\]

3. (i) (a) The uniform width \(h = 0.25\), we have:

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>0.25</th>
<th>0.5</th>
<th>0.75</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^3)</td>
<td>0</td>
<td>0.016</td>
<td>0.125</td>
<td>0.422</td>
<td>1</td>
</tr>
</tbody>
</table>

Using (9.1):

\[\int_0^1 x^3 dx \approx \frac{0.25}{2} \left[ 0 + 2(0.016 + 0.125 + 0.422) + 1 \right] = 0.266\]

(b) Similarly with \(h = 0.125\) we only have to evaluate \(x^3\) for \(x = 0.125, \ 0.375, \ 0.625\) and \(0.875\) because the others have been evaluated in the above table:

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>0.125</th>
<th>0.25</th>
<th>0.375</th>
<th>0.5</th>
<th>0.625</th>
<th>0.75</th>
<th>0.875</th>
<th>1.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x^3)</td>
<td>0</td>
<td>0.002</td>
<td>0.016</td>
<td>0.053</td>
<td>0.125</td>
<td>0.244</td>
<td>0.422</td>
<td>0.670</td>
<td>1</td>
</tr>
</tbody>
</table>

Applying (9.1):

\[\int_0^1 x^3 dx \approx \frac{0.125}{2} \left[ 0 + 2(0.002 + 0.016 + 0.053 + 0.125 + 0.244 + 0.422 + 0.670) + 1 \right]
   \[= 0.254\]

(8.1) \[\int x^n dx = x^{n+1}/n + 1\]

(9.1) \[\int_a^b y dx \approx \frac{h}{2} \left[ y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n \right]\]
(ii) Exact value is found by using (8.1):

$$\int_0^1 x^3 dx = \left[\frac{x^4}{4}\right]_0^1 = \frac{1}{4} = 0.25$$

(iii) (a) % error = \frac{0.25 - 0.266}{0.25} \times 100 = -6.4\% \quad \text{(using 4 intervals)}

(b) % error = \frac{0.25 - 0.254}{0.25} \times 100 = -1.6\% \quad \text{(using 8 intervals)}

(iv) As the number of interval increases so the accuracy of the estimation increases.

4. Applying (9.1) with \( h = 1 \) and \( y_0 = 2.1, \ y_1 = 9.56, \ y_2 = 11.36, \ y_3 = 12.08, \ y_4 = 12.98 \) and \( y_5 = 13.76 \):

Distance = \int_0^5 vdt \approx \frac{1}{2} [2.1 + 2(9.56 + 11.36 + 12.08 + 12.98) + 13.76]

= 53.91 m

5. Cross-sectional area is found by using the trapezium rule (9.1):

Cross-sectional area \approx \frac{1.5}{2} \left[ 0 + 2 \left( 1.04 + 1.65 + 3.10 + 4.66 + 4.12 \right) \right] + 0

= 33.975 m^2

Volume/sec \approx 33.975 \times 2.05 = 69.65 m^3 / s.

6. Using (9.1) with \( h = 0.1 \) and the \( y \) values given by the second row of the table:

$$\int_0^{0.6} vdt \approx \frac{0.1}{2} \left[ 4 + 2(3.92 + 3.86 + 3.77 + 3.61 + 3.52) + 3.41 \right]

= 2.24 \ Vs

\bar{v} = \frac{1}{0.6} \int_0^{0.6} vdt = \frac{2.24}{0.6} = 3.73 \ V$$

(9.1) \quad \int_a^b ydx \approx \frac{h}{2} [y_0 + 2(y_1 + y_2 + \ldots + y_{n-1}) + y_n]