Complete solutions to Exercise 14(a)

1. Integrate each function twice and substitute the given conditions to obtain the following:
   (a) \( s = -4.9t^2 \)
   (b) \( s = 1.63t^3 \)
   (c) \( s = ut + \frac{1}{2}at^2 \)

2. (a) Characteristic equation is given by\[ m^2 + 5m + 6 = 0 \]
   \[ (m + 3)(m + 2) = 0 \]
   \[ m_1 = -3, \ m_2 = -2 \]
   Since we have distinct roots so by (14.4) we have
   \[ y = Ae^{-3x} + Be^{-2x} \]

(b) Characteristic equation is \( m^2 + 4m + 4 = 0 \)
   \[ (m + 2)^2 = 0, \ \text{hence} \ m = -2 \ [\text{Equal Roots}] \]
   Substituting \( m = -2 \) into, (14.5), \( y = (A + Bx)e^{-2x} \) gives
   \[ y = (A + Bx)e^{-2x} \]

(c) Characteristic equation is \( m^2 - 2m + 4 = 0 \).
   Using the quadratic formula (1.16) with \( a = 1, \ b = -2 \) and \( c = 4 \) gives
   \[ m = \frac{2 \pm \sqrt{4 - 16}}{2} = 1 \pm j \frac{\sqrt{12}}{2} \]
   \[ m = 1 \pm j \sqrt{3} \]
   Putting \( \alpha = 1 \) and \( \beta = \sqrt{3} \) into (14.6) gives
   \[ y = e^{x} \left[ A \cos(\sqrt{3}x) + B \sin(\sqrt{3}x) \right] \]

3. We will just state the characteristic equation and then the solution.
   (a) We have
   \[ m^2 + 3m + 2 = 0 \]
   \[ (m + 2)(m + 1) = 0 \]
   \[ m_1 = -2, \ m_2 = -1 \ [\text{Distinct Roots}] \]
   By (14.4) the general solution is \( y = Ae^{-2x} + Be^{-x} \)

(1.16) \[ m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
(14.4) If \( m_1 \) and \( m_2 \) then \( y = Ae^{m_1x} + Be^{m_2x} \)
(14.5) Equal roots \( m \) then \( y = (A + Bx)e^{mx} \)
(14.6) If \( m = \alpha \pm j\beta \) then \( y = e^{\alpha x} \left[ A \cos(\beta x) + B \sin(\beta x) \right] \)
(b) The characteristic equation is given by
By (14.4) the general solution is \( y = Ae^{6x} + Be^x \)

(c) We have \( m^2 - 6m + 9 = 0 \)
\[
(m-3)^2 = 0 \quad \text{gives} \quad m_1 = m_2 = 3 \quad \text{[Equal Roots]}
\]

\( y = (A+3x)e^{3x} \)

(d) We have \( m^2 - 4m + 5 = 0 \)
\[
m = \frac{4 \pm \sqrt{16 - 20}}{2} = 2 \pm j \quad \text{[Complex Roots]}
\]

Using (14.6) with \( \alpha = 2 \) and \( \beta = 1 \) gives \( y = e^{2x} [A \cos(x) + B \sin(x)] \).

(e) We have \( m^2 - 8m + 16 = 0 \)
\[
(m-4)^2 = 0 \quad \text{gives equal roots} \quad m = 4
\]

By (14.5) we have the general solution \( y = (A+Bx)e^{4x} \)

4. (a) Characteristic equation is
\[
m^2 + 6m + 9 = 0
\]
\[
(m+3)^2 = 0, \quad m = -3
\]

Since we have equal roots, \( m = -3 \), so by (14.5)
\[
y = (A+Bx)e^{-3x}
\]

(b) Characteristic equation is
\[
10m^2 + 50m + 250 = 0
\]
\[
m^2 + 5m + 25 = 0 \quad \text{[Dividing by 10]}
\]

Putting \( a = 1, \quad b = 5 \) and \( c = 25 \) into (1.16) gives
\[
m = \frac{-5 \pm \sqrt{25 - (4 \times 25)}}{2} = \frac{-5 \pm \sqrt{25(1-4)}}{2}
\]
\[
= \frac{-5 \pm \sqrt{25 \sqrt{-3}}}{2} = \frac{-5 \pm 5\sqrt{3}}{2} = m \quad \text{[Complex Roots]}
\]

(1.16)
\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

(14.4) If \( m_1 \) and \( m_2 \) then \( y = Ae^{m_1x} + Be^{m_2x} \)

(14.6) If \( m = \alpha \pm j\beta \) then \( y = e^{\alpha x} [A \cos(\beta x) + B \sin(\beta x)] \)

(14.8) If \( r^2 + k^2 = 0 \) then \( y = A \cos(kx) + B \sin(kx) \)
Putting $\alpha = -\frac{5}{2}$ and $\beta = \frac{5\sqrt{3}}{2}$ into (14.6) gives

$$y = e^{-\frac{5}{2}x} \left[ A \cos \left( \frac{5\sqrt{3}}{2} x \right) + B \sin \left( \frac{5\sqrt{3}}{2} x \right) \right]$$

(c) Characteristic equation is

$$-m^2 - 3m + 8 = 0$$

$$m^2 + 3m - 8 = 0$$

[Multiplying by $-1$]

Putting $a = 1$, $b = 3$ and $c = -8$ into (1.16)

$$m = \frac{-3 \pm \sqrt{9 + 32}}{2} = -\frac{3}{2} \pm \frac{\sqrt{41}}{2}$$

$$m_1 = \frac{-3 + \sqrt{41}}{2}, \quad m_2 = \frac{-3 - \sqrt{41}}{2}$$

By (14.4) $y = Ae^{m_1x} + Be^{m_2x}$ where $m_1$ and $m_2$ are as above.

(1.16) $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(14.4) If $m_1$ and $m_2$ then $y = Ae^{m_1x} + Be^{m_2x}$