### Complete solutions to Exercise 1(f)

1. a. \(4(x + y + z)\)  
   b. \(8x(1 + y)\)  
   c. \(2(x - 2y)\)  
   d. \(x(3 - 2x)\)

e. Since \(x\) is common in both terms therefore we have  
   \[x^2 - xy = x(x - y)\]

f. What is common in \(16x + 4x^2\)?
   Since \(16x = 4 \times 4x\) therefore  
   \[16x + 4x^2 = (4 \times 4x) + 4x^2 = 4x(4 + x)\]

g. What is common between the two terms in the expression \(9x^2 - 27x^3\)?
   The numbers 9 and 27 have 9 in common and \(x^2\) and \(x^3\) have \(x^2\) in common. Hence  
   \[9x^2 - 27x^3 = 9x^2 - (9x^2 \times 3x)\]
   \[= 9x^2(1 - 3x)\]

2. Taking out the common factor in each term:
   (a) \(s = t(u + \frac{1}{2}at)\)  
   (b) \(F = \frac{m}{l}(v_2 - v_1)\)  
   (c) \(F = \rho AV_1v_2 - \rho AV_1v_1 = \rho AV_1(v_2 - v_1)\)

3. We have \(S = \pi r\left[r + \left(r^2 + h^2\right)^{1/2}\right]\)

4. Similar to Examples 24 and 25.
   (a) \((x + 5)(x + 2)\)  
   (b) \((x + 4)(x + 1)\)  
   (c) \((x - 4)(x - 1)\)  
   (d) \((x + 2)(x - 6)\)
   (e) \((2x - 1)(x + 1)\)  
   (f) \((x - 4)(x + 1)\)  
   (g) \((3x + 5)(7x - 2)\)
   (h) How do we factorize \(6x^2 + x - 12\)?

5. We have (a) \(x^2 - 2x + 1 = (x - 1)^2\)  
   (b) \(x^2 + 2x + 1 = (x + 1)^2\)
   (c) How do we factorize the given quadratic \(x^2 - 36\)?
   \(x^2 - 36\) is the difference of two squares which means we can use  
   \[(1.15)\ a^2 - b^2 = (a - b)(a + b)\]
   Hence we have \(x^2 - 36 = x^2 - 6^2 = (x - 6)(x + 6)\).
   (d) Similarly by applying (1.15) and writing \(\sqrt{7}\)^2 = 7 we have  
   \[x^2 - 7 = x^2 - (\sqrt{7})^2 = (x - \sqrt{7})(x + \sqrt{7})\]
   (e) This expression \(4x^2 + 12x + 9\) is more difficult to factorize:

6. Use (1.15) for both cases:
   (a) \(Z^2 - R^2 = (Z - R)(Z + R)\)  
   (b) \(\omega^2L^2 - \frac{1}{\omega^2C^2} = \left(\omega L - \frac{1}{\omega C}\right)\left(\omega L + \frac{1}{\omega C}\right)\)

7. \(F = \frac{2(V_s - V)}{V_s^2 - V^2} = \frac{2V(V_s - V)}{(V_s - V)(V_s + V)} = \frac{2V}{V_s + V}\)  
   \[(1.15)\ a^2 - b^2 = (a - b)(a + b)\]
   (Cancelling the common term \((V_s - V)\) in the numerator and denominator).
8. (a) Factorizing

\[
\frac{3wLx^2}{6EI} - \frac{wx^3}{6EI} = \frac{3wLx^2}{6EI} - \frac{wx^3}{6EI} = \frac{wx^2}{6EI} (3L - x)
\]

taking out the common factor

(b) In a similar manner

\[
\frac{wLx^3}{4EI} - \frac{3wx^4}{8EI} = \frac{2wLx^3}{8EI} - \frac{3wx^4}{8EI} = \frac{wx^3}{8EI} (2L - 3x)
\]

(c) Also

\[
\frac{wx^4}{24EI} - \frac{wLx^2}{12EI} + \frac{wL^2x^2}{24EI} = \frac{wx^2x^2}{24EI} - \frac{2wLx^2x}{24EI} + \frac{wL^2x^2}{24EI}
\]

\[
= \frac{wx^2}{24EI} (x^2) - \frac{wx^2}{24EI} (2Lx) + \frac{wx^2}{24EI} L^2
\]

\[
= \frac{wx^2}{24EI} (x^2 - 2Lx + L^2)
\]

How do we factorize the bracket term, \(x^2 - 2Lx + L^2\)?

We can use, (1.14), \(a^2 - 2ab + b^2 = (a - b)^2\)

\[x^2 - 2Lx + L^2 = (x - L)^2\]

So we have

\[
\frac{wx^4}{24EI} - \frac{wLx^2}{12EI} + \frac{wL^2x^2}{24EI} = \frac{wx^2}{24EI} (x - L)^2
\]

9. Very similar to **EXAMPLE 26**.

10. We have \(x\) and \(w\) which is common in every term of the numerator and \(4EI\) common on the denominator (4 goes into 8, 12 and 24). Hence

\[
y = \frac{wx}{4EI} \left( \frac{x^2}{3} - \frac{Lx}{2} + \frac{L^2}{6} \right)
\]

\[
= \frac{wx}{4EI} \left( \frac{x^2}{3} - \frac{Lx}{2} + \frac{L^2}{6} \right) \quad (†)
\]

How do we handle the terms inside the bracket \(\frac{x^2}{3} - \frac{Lx}{2} + \frac{L^2}{6}\)?

We need to determine the Lowest Common Multiple of 2, 3 and 6. Clearly it is 6. So

\[
\frac{x^2}{3} - \frac{Lx}{2} + \frac{L^2}{6} = \frac{2x^2}{6} - \frac{3Lx}{6} + \frac{L^2}{6}
\]

\[
= \frac{2x^2 - 3Lx + L^2}{6}
\]

Substituting this into (†) gives
\[ y = \frac{wx}{4EI} \left( \frac{2x^2 - 3lx + l^2}{6} \right) \]
\[ = \frac{wx}{24EI} (2x^2 - 3lx + l^2) \]

We need to factorize the terms in the bracket, \( 2x^2 - 3lx + l^2 = (2x - l)(x - l) \)

Hence our result: \( y = \frac{wx}{24EI} (2x - l)(x - l) \)

11. (a) We have \( N = \frac{Z_0 + \frac{1}{2} Z_1}{Z_0 - \frac{1}{2} Z_1} \), multiply the numerator and denominator by 2:

\[ N = \frac{2Z_0 + Z_1}{2Z_0 - Z_1} \]

Multiplying both sides by \( 2Z_0 - Z_1 \) and expanding gives

\[ (2Z_0 - Z_1)N = 2Z_0 + Z_1 \]
\[ 2Z_0N - Z_1N = 2Z_0 + Z_1 \]

Collecting the \( Z_0 \) terms on the Left Hand Side and \( Z_1 \) terms on the Right Hand Side:

\[ 2Z_0N - 2Z_0 = Z_1N + Z_1 \]

Factorizing:

\[ 2Z_0 (N - 1) = Z_1 (N + 1) \]

Thus

\[ Z_1 = \frac{2Z_0 (N - 1)}{N + 1} = 2Z_0 \left( \frac{N - 1}{N + 1} \right) \]

(b) We have

\[ Z_1 (N - 1)^2 + 2Z_0 (N^2 - 1) = Z_1 (N + 1)^2 \]
\[ 2Z_0 (N^2 - 1) = Z_1 \left[ (N + 1)^2 - (N - 1)^2 \right] \]
\[ = 4NZ_1 \]

Hence

\[ Z_1 = \frac{2Z_0 (N^2 - 1)}{4N} = \frac{Z_0 (N^2 - 1)}{2N} = Z_0 \left( \frac{N^2 - 1}{2N} \right) \]