Complete solutions to Exercise 1(g)

1. (a) \( x = \frac{1}{2} \)
(b) We need to factorize \( x^2 + 5x + 6 = 0 \). How?
\[ x^2 + 5x + 6 = (x + 3)(x + 2) = 0 \]
Therefore \( x = -2, \ x = -3 \).
(c) Factorizing \( x^2 - 10x + 21 \) gives
\[ x^2 - 10x + 21 = (x - 3)(x - 7) = 0 \]
This means that \( x = 3, \ x = 7 \).
(d) Factorizing the given quadratic \( 6x^2 - 13x - 5 \) gives
\[ 6x^2 - 13x - 5 = (2x - 5)(3x + 1) = 0 \]
We have
\[ (3x + 1) = 0 \text{ or } (2x - 5) = 0 \]
\( 3x = -1 \text{ or } 2x = 5 \)
\( x = -\frac{1}{3} \text{ or } x = \frac{5}{2} \)
(e) In a similar manner we have \( 5x^2 + 14x - 3 = (5x - 1)(x + 3) = 0 \). Solving this
\[ (5x - 1)(x + 3) = 0 \text{ implies that } x = \frac{1}{5}, \ x = -3 \]
(f) How do we factorize \( x^2 - 1 \)?
Use the difference of two squares (1.15) \( a^2 - b^2 = (a - b)(a + b) \):
\[ x^2 - 1 = (x - 1)(x + 1) = 0 \]
Therefore \( x = 1, \ x = -1 \).
(g) Similarly we have
\[ x^2 - 2x + 1 = (x - 1)^2 = 0 \]
This means \( x = 1 \).
(h) The factorization in this case is more difficult:
\[ 4x^2 + 8x + 4 = (2x + 2)(2x + 2) = (2x + 2)^2 = 0 \]
How do we solve the equation \( (2x + 2)^2 = 0 \)?
\[ (2x + 2)^2 = 0 \text{ implies } 2x + 2 = 0 \text{ which gives } x = -1 \]
Our solution is \( x = -1 \).
(i) How do we solve the given quadratic equation \( 3x^2 + 9x + 6 = 0 \)?
By taking the elementary step of dividing this equation by 3 we have
\[ x^2 + 3x + 2 = 0 \]
It is easier to factorize \( x^2 + 3x + 2 = 0 \):
\[ x^2 + 3x + 2 = (x + 1)(x + 2) = 0 \]
Therefore \( x = -1, \ x = -2 \).
(j) Again life is easier if we divide the given quadratic \( -2x^2 + 6x - 4 = 0 \) by \(-2\):
\[ x^2 - 3x + 2 = 0 \]
How do we solve this quadratic \( x^2 - 3x + 2 = 0 \)?
Factorizing gives \( x^2 - 3x + 2 = (x - 1)(x - 2) = 0 \) which implies \( x = 1, \ x = 2 \).
2. Substituting \( v = 35 \), \( a = 3.2 \) and \( s = 187 \) into \( u^2 + 2as = v^2 \) gives
\[
\begin{align*}
u^2 + (2 \times 3.2 \times 187) &= 35^2 \\
u^2 &= 35^2 - (2 \times 3.2 \times 187) = 28.2 \\
u &= \sqrt{28.2} = 5.31 \text{ m/s (2 d.p.)}
\end{align*}
\]

3. Factorizing : 
\[
2x^2 - 3xL + L^2 = (2x - L)(x - L) \\
\Rightarrow 2x - L = 0 \text{ or } x - L = 0.
\]
Thus 
\[
x = \frac{L}{2} \text{ or } x = L.
\]

4. We have \( 4.3t^2 + 1.9t = 50 \). Rearranging
\[
4.3t^2 + 1.9t - 50 = 0
\]
Using (1.16) \( t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \) with \( a = 4.3 \), \( b = 1.9 \) and \( c = -50 \) gives
\[
\begin{align*}
t &= \frac{-1.9 \pm \sqrt{1.9^2 - (4 \times 4.3 \times (-50))}}{2 \times 4.3} \\
&= \frac{-1.9 \pm \sqrt{3.61 + 1860}}{8.6} \\
&= \frac{-1.9 \pm 29.387}{8.6} \\
&= \frac{-1.9 + 29.387}{8.6} \text{ or } \frac{-1.9 - 29.387}{8.6}
\end{align*}
\]
Since \( t \geq 0 \) so \( t = 3.20 \) s (2 d.p.).

5. Let \( w \) be the width then \( w = \ell - 5 \) and
\[
\ell (\ell - 5) = 84 \\
\ell^2 - 5\ell - 84 = 0 \\
(\ell - 12)(\ell + 7) = 0 \\
\ell = 12 \text{ or } \ell = -7
\]
Dimensions are \( \ell = 12 \) m and \( w = 12 - 5 = 7 \) m.

6. Putting \( M = 0 \) gives 
\[
-20x^2 - 500x + 3000 = 0
\]
Dividing by \(-20\): 
\[
x^2 + 25x - 150 = 0
\]
Factorizing
\[
(x - 5)(x + 30) = 0
\]
Hence \( x = 5 \) gives \( M = 0 \).
7. \[ M = \frac{15}{8} x - \frac{29}{4} \left( x - \frac{1}{2} \right)^2 = \frac{15}{8} x - \frac{58}{8} \left( x - \frac{1}{2} \right)^2 \]
\[ = \frac{1}{8} \left[ 15x - 58 \left( x^2 - x + 0.25 \right) \right] \]
\[ = \frac{1}{8} \left[ 15x - 58x^2 + 58x - 14.5 \right] \]
\[ = \frac{1}{8} \left[ 73x - 58x^2 - 14.5 \right] \]

With \( M = 0 \) we have
\[ -58x^2 + 73x - 14.5 = 0 \]
Multiplying by -1 gives \( 58x^2 - 73x + 14.5 = 0 \)
Putting \( a = 58 \), \( b = -73 \) and \( c = 14.5 \) into (1.16) \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \] gives
\[ x = \frac{73 \pm \sqrt{(-73)^2 - (4 \times 58 \times 14.5)}}{2 \times 58} \]
\[ = \frac{1.011}{2} \text{ or } 0.247 \]
\( x = 1.01 \text{ m (2 d.p.) or } x = 0.25 \text{ m (2 d.p.)} \)

8. Putting \( h = 0 \) gives \(-4.9t^2 + 55t + 12 = 0\). So solving produces \( t = 11.44 \text{s} \)

9. We have \( h = 0 \) because \( h \) is the height above ground, so
\[ -\frac{1}{2} gt^2 + ut + h_0 = 0 \]
Multiplying by -1:
\[ \frac{1}{2} gt^2 - ut - h_0 = 0 \]
Putting \( a = \frac{1}{2} g, \ b = -u \) and \( c = -h_0 \) into (1.16) gives
\[ t = \frac{u \pm \sqrt{(-u)^2 - \left( 4 \times \frac{1}{2} g \times (-h_0) \right)}}{2 \times \frac{1}{2} g} \]
\[ t = \frac{u \pm \sqrt{u^2 + 2gh_0}}{g} \]

(1.14) \( (a-b)^2 = a^2 - 2ab + b^2 \)
(1.16) \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
10. We have

\[
-\frac{T}{2wL^2} = \frac{4kL^{\frac{5}{2}} - 3DL^\frac{1}{2} - 3kL^\frac{5}{2}}{2L^3}
\]

\[
= \frac{kL^{\frac{1}{2}} - 3DL^\frac{1}{2}}{2L^3}
\]

\[
\implies \frac{kL^{\frac{1}{2}} - 3DL^\frac{1}{2}}{2} + \frac{T}{2wL^2} = 0
\]

Multiplying by \(2L^\frac{5}{2}\) gives a quadratic

\[
kL^\frac{1}{2} - 3DL^\frac{1}{2} + \frac{T}{w} L = 0
\]

\[
kL^\frac{1}{2} + \frac{T}{w} L - 3D = 0
\]

How do we solve this quadratic?

Use (1.16) with \(a = k\), \(b = \frac{T}{w}\) and \(c = -3D\):

\[
L = \frac{-\frac{T}{w} \pm \sqrt{\frac{T^2}{w^2} + (4k \times 3D)}}{2k}
\]

\[
= \frac{-\frac{T}{w} \pm \sqrt{T^2 + (4k \times 3D)w^2}}{2k}
\]

\[
= \frac{-\frac{T}{w} \pm \sqrt{T^2 + 12kDw^2}}{2k}
\]

\[
L = \frac{-T \pm \sqrt{T^2 + 12kDw^2}}{2kw}
\]

(1.16)

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]