Complete solutions to Exercise 1(h)

1. (a) Similar to EXAMPLE 30, \( x = 1, \ y = 1 \).

(b) We add the two given simultaneous equations:

\[
\begin{align*}
\text{Equation(1)} & : \quad x + y = 2 \\
\text{Equation(2)} & : \quad x - y = 0
\end{align*}
\]

We have \( 2x = 2 \) which gives \( x = 1 \). \textit{How do we find the value of the other unknown,} \( y \)?

Substitute \( x = 1 \) into the first equation \( x + y = 2 \):

\[
1 + y = 2 \quad \text{which gives} \quad y = 1
\]

Hence \( x = 1 \) and \( y = 1 \) is the solution.

(c) We can label each linear equation

\[
\begin{align*}
\text{Equation(1)} & : \quad 2x - 3y = 5 \\
\text{Equation(2)} & : \quad x - y = 2
\end{align*}
\]

\textit{How do we eliminate one of the unknowns?}

Eliminate \( x \) by multiplying equation (***) by 2 and subtracting from equation (*):

\[
\begin{align*}
2x - 3y &= 5 \quad (*) \\
2x - 2y &= 4 \quad \text{[Multiplying (***) by 2]}
\end{align*}
\]

\[
0 - y = 1
\]

Hence \( y = 1 \). \textit{How do we determine} \( x \)?

By substituting \( y = 1 \) into the given equation (**):

\[
x - (1) = 2
\]

\[
x + 1 = 2 \quad \text{which gives} \quad x = 1
\]

The solution is \( x = 1 \) and \( y = 1 \).

(d) We label each equation:

\[
\begin{align*}
\text{Equation(1)} & : \quad 2x - 3y = 35 \\
\text{Equation(2)} & : \quad x - y = 2
\end{align*}
\]

We need to eliminate one of the unknowns. \textit{Which one?}

To make life easier it is better to eliminate \( x \). \textit{How?}

Multiply equation (***) by 2 and subtract from equation (***):

\[
\begin{align*}
2x - 3y &= 35 \quad ($) \\
2x - 2y &= 4 \quad \text{[Multiplying (***) by 2]}
\end{align*}
\]

\[
0 - y = 31
\]

We have \( y = -31 \). \textit{How do we find} \( x \)?

Substitute \( y = -31 \) into the given equation (***):

\[
x - (-31) = 2
\]

\[
x + 31 = 2 \quad \text{which gives} \quad x = -29
\]

The solution is \( x = -29 \) and \( y = -31 \).

(e) We label each equation:

\[
\begin{align*}
\text{Equation(1)} & : \quad 5x - 7y = 2 \\
\text{Equation(2)} & : \quad 9x - 3y = 6
\end{align*}
\]

\text{Equation(1)}: \( 5x - 7y = 2 \) \hspace{1cm} \text{Equation(2)}: \( 9x - 3y = 6 \)
We need to eliminate one of the unknowns. Which one?
Eliminate \( y \). How?

Multiply equation (†) by 3 and multiply (††) by 7:
\[
\begin{align*}
15x - 21y &= 6 & [\text{Multiplying (†) by 3}] \\
63x - 21y &= 42 & [\text{Multiplying (††) by 7}]
\end{align*}
\]

To eliminate \( y \) we subtract these equations
\[
\begin{align*}
63x - 21y &= 42 \\
-15x - 21y &= 6 \\
\hline
48x - 0 &= 36 & [\text{Subtracting}]
\end{align*}
\]

From \( 48x = 36 \) we have \( x = \frac{36}{48} = \frac{3}{4} \). How do we find \( y \)?

Substitute \( x = \frac{3}{4} \) into the given equation (†):

\[
\begin{align*}
5\left(\frac{3}{4}\right) - 7y &= 2 \\
\frac{15}{4} - 7y &= 2 \\
7y &= \frac{15}{4} - 2 = \frac{7}{4}
\end{align*}
\]

How do find \( y \) from \( 7y = \frac{7}{4} \) ?

Dividing both sides by 7:
\[
y = \frac{7}{4(7)} = \frac{1}{4}
\]

The solution is \( x = \frac{3}{4} \) and \( y = \frac{1}{4} \).

(f) We have the equations
\[
\begin{align*}
\pi x - 5y &= 2 & (\ast) \\
\pi x - y &= 1 & (\ast\ast)
\end{align*}
\]

Subtracting these equations we have
\[
\begin{align*}
\pi x - 5y &= 2 \\
- \pi x + y &= 1 \\
\hline
0 - 4y &= 1
\end{align*}
\]

From the last line \(-4y = 1\) we have \( y = -\frac{1}{4} \). How do we determine \( x \)?

Substitute \( y = -\frac{1}{4} \) into (\ast\ast):
\[
\pi x - \left( -\frac{1}{4} \right) = 1
\]
\[
\pi x + \frac{1}{4} = 1 \quad \pi x = \frac{3}{4} \quad \text{which gives} \quad x = \frac{3}{4\pi}
\]

The solution is \( x = \frac{3}{4\pi} \) and \( y = -\frac{1}{4} \).

2. Putting \( E = 53, \ W = 120 \) into \( E = aW + b \) gives
\[
53 = 120a + b \quad (\dagger)
\]
and \( E = 45.5, \ W = 70 \) into \( E = aW + b \) gives
\[
45.5 = 70a + b \quad (\ddagger)
\]
Subtract (\ddagger) from (\dagger)
\[
7.5 = 50a
\]
Hence
\[
a = \frac{7.5}{50} = 0.15
\]
Substituting \( a = 0.15 \) into (\dagger) gives
\[
(120 \times 0.15) + b = 53
\]
\[
18 + b = 53
\]
\[
b = 35 \text{ N}
\]
Putting \( a = 0.15 \) and \( b = 35 \) into \( E = aW + b \) gives \( E = 0.15W + 35 \)

3. Putting \( t = 2, \ s = 33 \) into \( ut + \frac{1}{2}at^2 = s \) gives
\[
2u + \left( \frac{1}{2}a2^2 \right) = 33
\]
\[
2u + 2a = 33 \quad (\dagger)
\]
Putting \( t = 3, \ s = 64.5 \) into \( ut + \frac{1}{2}at^2 = s \) gives
\[
3u + \frac{3^2}{2}a = 64.5
\]
\[
3u + 4.5a = 64.5 \quad (\ddagger)
\]
Multiply (\dagger) by 3
\[
6u + 6a = 99 \quad (*)
\]
Multiply (\ddagger) by 2
\[
6u + 9a = 129 \quad (**)
\]
\((**)-(*)\) gives
\[
3a = 30
\]
\[
a = 10
\]
Substituting \( a = 10 \) into (\dagger)
\[
2u + 20 = 33
\]
\[
2u = 13
\]
\[
u = 6.5
\]
\[
a = 10 \text{ m/s}^2 \text{ and } u = 6.5 \text{ m/s}
\]

4. Opening up the brackets gives
\[
25I_1 - 25I_2 + 56I_1 = 2.225
\]
\[
17I_2 - 3I_1 + 3I_2 = 1.31
\]
Simplifying

\[ 81I_1 - 25I_2 = 2.225 \]
\[ -3I_1 + 20I_2 = 1.31 \]

Solving these gives \( I_1 = 50 \text{ mA} \) and \( I_2 = 73 \text{ mA} \)

5. We have

\[ \ell_o (1 + 55\alpha) = 20.11 \quad (\dagger) \]
\[ \ell_o (1 + 120\alpha) = 20.24 \quad (\ddagger) \]

(\dagger) divided by (\ddagger) gives

\[ \frac{\ell_o (1 + 55\alpha)}{\ell_o (1 + 120\alpha)} = \frac{20.11}{20.24} \]
\[ 20.24(1 + 55\alpha) = 20.11(1 + 120\alpha) \]
\[ 20.24 + 1113.2\alpha = 20.11 + 2413.2\alpha \]
\[ 20.24 - 20.11 = 2413.2\alpha - 1113.2\alpha \]
\[ 0.13 = 1300\alpha \]
\[ \alpha = \frac{0.13}{1300} = 1 \times 10^{-4} \]

Substituting \( \alpha = 1 \times 10^{-4} \) into (\ddagger) results in

\[ \ell_o \left( 1 + \left( 120 \times 1 \times 10^{-4} \right) \right) = 20.24 \]
\[ 1.012\ell_o = 20.24 \]
\[ \ell_o = 20 \]

Hence \( \ell_o = 20m \) and \( \alpha = 1 \times 10^{-4}/°C \)

6. From the first formula we have

\[ \frac{1}{R_1} = \left( 1.2 \times 10^{-3} \right) - \frac{1}{R_2} \quad (\dagger) \]

Substituting this into

\[ \frac{5}{R_1} + \frac{8}{R_2} = 5 \left( \frac{1}{R_1} \right) + \frac{8}{R_2} = 6.6 \times 10^{-3} \]

gives

\[ 5 \left( 1.2 \times 10^{-3} \right) - \frac{1}{R_2} + \frac{8}{R_2} = 6.6 \times 10^{-3} \]
\[ 6 \times 10^{-3} + \frac{3}{R_2} = 6.6 \times 10^{-3} \]
\[ \frac{3}{R_2} = \left( 6.6 \times 10^{-3} \right) - \left( 6 \times 10^{-3} \right) = 6 \times 10^{-4} \]
\[ R_2 = \frac{3}{6 \times 10^{-4}} = 5000 \]

Substituting \( R_2 \) into (\dagger) gives

\[ \frac{1}{R_1} = \left( 1.2 \times 10^{-3} \right) - \frac{1}{5000} = 0.001 \]
\[ R_1 = \frac{1}{0.001} = 1000 \]
\[ R_1 = 1kΩ \quad \text{and} \quad R_2 = 5kΩ \]
7. Using TABLE 1 and considering dimensions

\[ MLT^{-2} = (ML^a)^b(L^2)^c(LT^{-1})^c \]

\[ \equiv M^a L^{-3a} L^{2b} L^c T^{-c} \]

Using rules of indices

\[ MLT^{-2} = M^a L^{-3a + 2b + c} T^{-c} \]

by (1.5)

Equating powers of M, L and T respectively gives

1 = a

1 = −3a + 2b + c \quad (*)

−2 = −c so c = 2

Substituting \( a = 1, \ c = 2 \) into (*)

\[ −3 + 2b + 2 = 1 \]

\[ b = 1 \]

Substituting \( a = 1, \ b = 1 \) and \( c = 2 \) into \( F = K \rho^a A^b v^c \) gives

\[ F = K \rho A v^2 \]