**Complete solutions to Exercise 5(c)**

1. We use our calculator:
   
   (a) \( G = \ln \left( \frac{0.1}{0.01} \right) = 2.30 \text{ nep} \) 
   
   (b) \( G = \ln \left( \frac{10}{0.1} \right) = 4.61 \text{ nep} \) 
   
   (c) \( G = \ln \left( \frac{5}{15} \right) = -1.10 \text{ nep} \)

2. (a) \( G = \frac{1}{2} \ln \left( \frac{1}{0.2} \right) = 0.80 \text{ nep} \) 
   
   (b) \( G = \frac{1}{2} \ln(20) = 1.50 \text{ nep} \)

3. Remember \( 1 \text{ mV} = 1 \times 10^{-3} \text{ V} \).
   
   \[ G = 20 \log \left( \frac{1}{1 \times 10^{-3}} \right) = 20 \log(10^3) \Rightarrow (3 \times 20 \log(10)) = 60 \text{ dB} \]

4. We use \( G = 10 \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) \) with \( G = 5 \) and \( P_{\text{in}} = 0.1 \):
   
   \[ 10 \log \left( \frac{P_{\text{out}}}{0.1} \right) = 5 \]
   
   \[ \log \left( \frac{P_{\text{out}}}{0.1} \right) = \frac{5}{10} = 0.5 \]

   How do remove the log function?
   
   Taking exponentials to the base 10:
   
   \[ \frac{P_{\text{out}}}{0.1} = 10^{0.5} \]
   
   \[ P_{\text{out}} = 0.1 \times 10^{0.5} = 0.316 W \]

5. Since 10 is a common factor of the Right Hand Side we have:
   
   \[ G = 10 \left[ \log \left( p_1 \right) + \log \left( p_2 \right) + \log \left( p_3 \right) \right] \]
   
   \[ \Rightarrow 10 \log \left( p_1 \times p_2 \times p_3 \right) \]

   by (5.17)

6. Substituting \( P_{\text{out}} = I_{\text{out}}^2 R \) and \( P_{\text{in}} = I_{\text{in}}^2 R \) into \( G = 10 \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right) \) gives:
   
   \[ G = 10 \log \left( \frac{I_{\text{out}}^2 R}{I_{\text{in}}^2 R} \right) \]
   
   \[ = 10 \log \left( \frac{I_{\text{out}}}{I_{\text{in}}} \right)^2 \] (Cancelling \( R \)'s)
   
   \[ G \equiv 20 \log \left( \frac{I_{\text{out}}}{I_{\text{in}}} \right) \]

(5.17) \[ \log (A) + \log (B) = \log (A \times B) \]

(5.19) \[ \log (A^n) = n \log (A) \]
7. We have:

\[20 \log \left( \frac{I_{\text{out}}}{I_{\text{in}}} \right) = 20\]

\[\log \left( \frac{I_{\text{out}}}{I_{\text{in}}} \right) = 1 \quad \text{(Dividing by 20)}\]

\[\left( \frac{I_{\text{out}}}{I_{\text{in}}} \right) = 10^1 = 10\]

\[I_{\text{out}} = 10 \times I_{\text{in}} \quad \text{(Multiplying by } I_{\text{in}})\]

\[= 10 \times 1 \times 10^{-3} \quad \text{(because } I_{\text{in}} = 1 \times 10^{-3})\]

\[I_{\text{out}} = 0.01 A\]

8. By substituting \(p_{\text{out}} = \frac{V_{\text{out}}^2}{R}\) and \(p_{\text{in}} = \frac{V_{\text{in}}^2}{R}\) into \(G = 10 \log \left( \frac{p_{\text{out}}}{p_{\text{in}}} \right)\) gives:

\[G = 10 \log \left( \frac{V_{\text{out}}^2}{V_{\text{in}}^2} \right)\]

\[= 10 \log \left( \frac{V_{\text{out}}}{V_{\text{in}}} \right)^2\]

\[= 20 \log \left( \frac{V_{\text{out}}}{V_{\text{in}}} \right)\]

9. Let \(p = \frac{P_{\text{out}}}{P_{\text{in}}}\) then we have:

(a) \(10 \log(p) = 3\) gives \(\log(p) = \frac{3}{10} = 0.3\) (Dividing by 10)

Taking exponential to the base 10: \(p = 10^{0.3} = 2\)

(b) \(10 \log(p) = 10\). Thus \(\log(p) = 1:\)

\[p = 10^1 = 10\]

(c) Similarly \(10 \log(p) = 20\), \(\log(p) = 2:\)

\[p = 10^2 = 100\]

To find the voltage ratio, \(\frac{V_{\text{out}}}{V_{\text{in}}} = V\), we only have to take the square root of the \(p\) values, because \(V = \sqrt{p}\) from \(\log(p) = \log(V^2)\) or \(V^2 = p\):

(a) \(V^2 = 2\) gives \(V = \sqrt{2}\)

(b) \(V^2 = 10\) gives \(V = \sqrt{10}\)

(c) \(V^2 = 100\) gives \(V = 10\)

10. We have

\[9 - 10e^{-t} = 0\]

\[9 = 10e^{-t} \quad \text{(Rearranging)}\]

\[e^{-t} = 0.9 \quad \text{(Dividing by 10)}\]

\[(5.19) \quad \log \left( \frac{A^n}{A} \right) = n \log(A)\]
Taking natural logs, \( \ln \), of both sides gives:
\[
\ln(e^{-t}) = \ln(0.9)
\]
\[
-t \ln(e) = \ln(0.9)
\]
\[
-t = \ln(0.9)
\]
\[
t = -\ln(0.9) = 0.105\text{sec}
\]

11. We have:
\[
9(1 - e^{-0.1t}) = 3
\]
\[
1 - e^{-0.1t} = \frac{3}{9} = \frac{1}{3} \quad \text{(Dividing by 9)}
\]
\[
1 - \frac{1}{3} = e^{-0.1t} \quad \text{(Rearranging)}
\]
\[
e^{-0.1t} = \frac{2}{3}
\]

Taking natural logs, \( \ln \), of both sides gives \( \ln(e^{-0.1t}) = \ln\left(\frac{2}{3}\right) \), so by (5.13) we have:
\[
-0.1 \ln(e) = \ln\left(\frac{2}{3}\right) = -0.405
\]
\[
t = \frac{-0.405}{-0.1} = 4.05\text{sec}
\]

12. Applying the change of base rule, (5.23), in each case and rounding to 3 d.p. we have:
(a) \( \log_2(7.6) = \frac{\ln(7.6)}{\ln(2)} = 2.926 \)
(b) \( \log_{7.6}(2) = \frac{\ln(2)}{\ln(7.6)} = 0.342 \)
(c) \( \log_5(6.3) = \frac{\ln(6.3)}{\ln(7)} = 0.946 \)
(d) \( \log_{6.3}(7) = \frac{\ln(7)}{\ln(6.3)} = 1.057 \)

13. (a) \( \ln(\ln(1000)) = \ln(6.907) = 1.93 \)
(b) \( \ln(\ln(10^{10})) = \ln(23.026) = 3.14 \)
(c) \( \ln(\ln(10^{50})) = \ln(115.129) = 4.75 \)

The \( \ln(\ln) \) function makes a large number much smaller.

14. By **EXAMPLE 9** we have \( \log(PV^n) = \log(C) \). Taking exponentials to the base 10 gives:
\[
PV^n = C
\]

15. We have
\[
e^{x \ln(a)} = \left[e^{\ln(a)}\right]^{x} = a^x
\]

\[\text{(5.13)}\]
\[
\ln(A^n) = n \ln(A)
\]

\[\text{(5.23)}\]
\[
\log_a(N) = \frac{\log_b(N)}{\log_b(a)}
\]
16. By taking exponential of both sides we obtain
\[ e^{\ln(1+y)} = e^{x + x^2} \]
\[ 1 + y = e^x e^{x^2} \]
by (5.16) by (5.1)
\[ y = e^{x^2} e^x - 1 \]

17. (a) **How do we solve** \( 27^x = 3 \)?

Note that \( 27 = 3 \). Substituting this \( 27 = 3 \) we have
\[ (3^3)^x = 3 \]
\[ 3^{3x} = 3 \quad \text{[Using } (a^n)^m = a^{nm} \text{]} \]

This means that \( 3x = 1 \) which implies that \( x = \frac{1}{3} \).

(b) We need to solve \( 5 \ln(x^2 - 9) = 3 \). Dividing by 5 yields
\[ \ln(x^2 - 9) = 0.6 \]

Taking exponential of both sides gives
\[ x^2 - 9 = e^{0.6} = 1.822 \]

Adding 9 to both sides gives
\[ x^2 = 9 + 1.822 = 10.822 \]
\[ x = \pm \sqrt{10.822} = \pm 3.29 \]

(c) **How do we solve** \( \log_9(3^{x-1}) = x \)?

By using the definition of logs we have
\[ 3^{x-1} = 9^x \]
\[ 3^{x-1} = (3^2)^x = 3^{2x} \]

We have \( x - 1 = 2x \) implies that \( x = -1 \).

(d) Taking natural logs of \( 3^x = 18 \):
\[ \ln(3^x) = \ln(18) \]
\[ x \ln(3) = \ln(18) \] implies that \( x = \frac{\ln(18)}{\ln(3)} = 2.63 \)

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(5.1) \[ a^{m+n} = a^m a^n \]
(5.16) \[ e^{\ln(a)} = a \]
18. We have
\[ \theta_1 - \theta_2 = -\frac{Q}{2\pi kL} \left[ \ln(r_1) - \ln(r_2) \right] \]

\[ = \frac{Q}{2\pi kL} \left[ -\ln(r_1) + \ln(r_2) \right] \]

taking the negative sign inside the brackets

\[ = \frac{Q}{2\pi kL} \left[ \ln(r_2) - \ln(r_1) \right] \]

\[ = \frac{Q}{2\pi kL} \ln \left( \frac{r_2}{r_1} \right) \]  

by (5.12)

Rearranging gives
\[ \frac{2\pi kL(\theta_1 - \theta_2)}{\ln \left( \frac{r_2}{r_1} \right)} = Q \]

19. We have by rearranging:
\[ e^{11600\eta T} = \frac{i}{I_s} + 1 \]

Taking ln's of both sides gives:
\[ \frac{11600\eta T}{\eta T} = \ln \left( \frac{i}{I_s} + 1 \right) \]

\[ v = \frac{\eta T}{11600} \ln \left( \frac{i}{I_s} + 1 \right) \text{ Multiplying by } \frac{\eta T}{11600} \]

Substituting \( \eta = 2 \), \( i = 5 \times 10^{-3} \), \( I_s = 0.1 \times 10^{-6} \) and \( T = 330 \) gives
\[ v = \frac{2 \times 330}{11600} \times \ln \left( \frac{5 \times 10^{-3}}{0.1 \times 10^{-6}} + 1 \right) \]

\[ = 0.62 \text{ volts} \]

(5.12) \( \ln(A) - \ln(B) = \ln \left( \frac{A}{B} \right) \)