The order of operations

Calculate $3 - 2 \times 4$.

It may look simple enough, but there is a snag. There are two ways to proceed and they each give a different answer. You might have reasoned:

(i) subtract first to give $3 - 2 = 1$ and then calculate $1 \times 4 = 4$ so the answer is 4

or you might have said

(ii) multiply first, so $2 \times 4 = 8$, and $3 - 8 = -5$.

Which one did you do?

The answer depends on the order in which the calculations are performed – whether to subtract or multiply first – and so longer expressions may have many more than two alternative answers. This situation is unsatisfactory so we need some sort of rule which tells us the order in which to perform the operations.

The accepted rule is to multiply and divide first, performing calculations from left to right and then add and subtract, also from left to right. For the example above we should multiply first, so the second answer, $3 - 2 \times 4 = 3 - 8 = -5$, is correct.

Let's try $6 \times 2 \div 4 + 1$. The rule tells us to multiply and divide first, but there is both a multiplication and a division here, so we must work from left to right. The multiplication occurs before the division, so we multiply 6×2 first to give

$$12 \div 4 + 1,$$

then divide giving

$$3 + 1,$$

and finally do the addition,

$$= 4.$$

check these

Remember to perform calculations from left to right if there is more than one multiplication/division or more than one addition/subtraction.

$$\begin{array}{rccccccccc} & -2 & + & 7 & \times & 8 & \div & 2 & \times & 2 \\ = & -2 & + & & 56 & & \div & 2 & \times & 2 \\ = & -2 & + & & & & 28 & & \times & 2 \\ = & -2 & + & & & & & & 56 & = & 54 \end{array}$$

$$\begin{array}{rccccccc} & 50 & - & 32 & \div & 16 & \times & 2 \\ = & 50 & - & & 2 & & \times & 2 \\ = & 50 & - & & & & 4 & = & 46 \end{array}$$

Introducing brackets

Suppose we need to multiply $2 + 3$ by $4 - 2$. We cannot write this as

$$2 + 3 \times 4 - 2$$

because applying the order of operations rule would give

$$2 + 12 - 2 = 12.$$

To show that the numbers must be processed in a different order we use brackets.

When part of an expression must be evaluated first it must be enclosed in brackets. So the multiplication of $2 + 3$ by $4 - 2$ must be written $(2 + 3) \times (4 - 2)$. This indicates that we must first calculate $2 + 3$, and then calculate $4 - 2$, and finally multiply the results, $5 \times 2 = 10$.

As evaluating the expressions in brackets takes priority over anything else, we can extend the rule for the order of operations to brackets, multiply and divide, add and subtract.

For example, to evaluate $(9 - 2) \times 10 - (2 \times 3)$ we work out the brackets first to give

$$7 \times 10 - 6$$

then multiply to give

$$70 - 6$$

and finally subtract to obtain

$$= 64.$$

The order of operations in evaluating arithmetic expressions is

Brackets
Multiply and Divide (from left to right)
Add and Subtract (from left to right)

When multiplying by a bracket it is usual to omit the multiplication sign. So $(2 + 3) \times (4 - 2)$ is written $(2 + 3)(4 - 2)$ and $2 \times (4 + 7)$ is written $2(4 + 7)$.

check these

$$\begin{aligned}
& 6 \quad (4-6) \quad (4+2) \quad + \quad 3 \\
= \; & 6 \times (-2) \times 6 \quad + \quad 3 \\
= \; & (-12) \times 6 \quad + \quad 3 \\
= \; & (-72) \quad + \quad 3 \quad = -69
\end{aligned}$$

$$\begin{aligned}
& 36 \quad \div \quad (4\times3) \quad - \quad 5 \\
= \; & 36 \quad \div \quad 12 \quad - \quad 5 \\
= \; & 3 \quad - \quad 5 \quad = -2
\end{aligned}$$

Quotients can be written using brackets. For instance,

$$\frac{80-20}{10-5}$$

can be written $(80 - 20) \div (10 - 5)$ because everything above the line of a quotient is divided by everything below the line.

Sometimes more than one layer of brackets is necessary. Do not be put off by this. You will find that you have to work out the inside brackets first and then proceed outwards. For instance

$$\begin{aligned}
(6(1 + 4)) \div 10 \; & = (6 \times 5) \div 10 \\
& = 30 \div 10 \\
& = 3.
\end{aligned}$$

Authors and lecturers are sometimes helpful and use different symbols for different 'layers' of brackets. For instance $\{[(2 + 3) + 5] \times 7\}$. As you will not always encounter this we have often used only one symbol in our work.

check this

$$\begin{aligned}
& 10 \times (2 \quad + \quad (6\div3) \times 4) \\
= \; & 10 \times (2 \quad + \quad 2 \quad \times \quad 4) \\
= \; & 10 \times (2 \quad + \quad \quad 8) \\
= \; & 10 \times \quad \quad 10 \quad \quad = 100
\end{aligned}$$

In practice, brackets are often used to clarify expressions when they are not strictly essential.

work card 3

1. Evaluate the following:
   - **a.** $(20 - 5) \times (4 - 2)$
   - **b.** $2 + (10 \div 5) \times 3$
   - **c.** $2 \times (10 \div 5) \times 2$
   - **d.** $2 \times 10 \div (5 \times 2)$
   - **e.** $\dfrac{10 + 20}{4 - 2}$
   - **f.** $2 \times 2 \times (27 \div 3) + (1 - 20)$
   - **g.** $(4 \times 2 \times 2) + (5 \times (-1))$
2. Evaluate the following:
   - **a.** $1 + 3 \times (4 + (8 \div 2))$
   - **b.** $((50 \div 25) \times 8 \div (7 - 3)) \times 3$
   - **c.** $12 - (4\,(8 \times (6 - 4)) - 5)$
   - **d.** $\dfrac{18 \times (2 - 3 \times 4)}{(4 + 14)}$
   - **e.** $\left(\dfrac{18 \div 3}{(4 \times 3) - 36}\right) \times 4$
   - **f.** $(-10) \times \left(\left(\dfrac{100}{25} \times 2\right) + (60 \div 20)\right) + 1$
   - **g.** $1 + \left(\dfrac{2 \times (2 + 4 \times 5)}{(2 \times 11) \div (22 \div 2)}\right)$

Solutions:

1. **a.** 30 **b.** 8 **c.** 8 **d.** 2 **e.** 15 **f.** 17 **g.** 11
2. **a.** 25 **b.** 12 **c.** –47 **d.** –10 **e.** –1 **f.** –109 **g.** 23

assessment 3

1. Evaluate:
   - **a.** $(40 \div 2) + (3 \times 4)$
   - **b.** $-5 + (-3) \times 2 + 1$
   - **c.** $6 \div (3 \times 2) + 4$
   - **d.** $3 \times 3 \times (6 \div 2) + 3$
   - **e.** $3 \times 3 \times 6 \div 2 + 3$
   - **f.** $\dfrac{3 + 6}{5 - 2}$
   - **g.** $\dfrac{10 - 2}{3 + 1} \div (-2)$
2. Evaluate:
   - **a.** $(21 \div 7) + (50 \div (5 \times 2) + 1)$
   - **b.** $\dfrac{77 \div 11}{108 \div (-3) \times 4}$
   - **c.** $\left(\dfrac{40}{6 + 2}\left(3 + \dfrac{5 - (21 \div 7)}{6 - 4} \times \dfrac{48}{16}\right)\right) + 5$
   - **d.** $\left(20{,}000 / \left((1000 - (2 \times 5 \times 50)) \times \left(\dfrac{100}{50}\right)\right)\right) - 2$
   - **e.** $\dfrac{30 + 3 \times 3}{10 + 3} + 5 \times \dfrac{10 + 100}{11}$

4 Using a calculator

test box 4

1. Use a calculator to evaluate:

   $\dfrac{500}{1.2(20 + 34)}$ to 6 decimal places.
2. Express the following to 3 decimal places:

   1.9755 10,002.9999 209.452 12.73 0.000123456
3. Express the numbers in question 2 to 3 significant figures.
4. Express the following in scientific notation:

   12,000,000 0.00001254
5. By estimating roughly, say whether you think the following is correct:

   $\dfrac{515 \times 6.1}{200} = 7.1$

INDEX